48. Observed Value of the Autocorrelation Function

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1. Introduction

The characteristics of the time series are given by its autocorrelation function or its Fourier transform, spectral density. Then, it is necessary to get its autocorrelation function by the observation for the study of time series. However, it is inevitable for some errors to be introduced in spite of our efforts to avoid them. In the measurement of the autocorrelation function, some sorts of fluctuations are considered, one of which is that of delay time. Other fluctuations, for example, that of amplitude seems to be based upon the mechanism of the instrument, but the author is not qualified to judge it, because he cannot see this instrument in his neighbourhood. The fluctuation of the delay time is very interesting from the theoretical standpoint and we can understand it from the uncertainty relation between time and frequency. The observed value of the spectrum has the definite relation with the mathematical spectrum and is correctable by the idealized experiment. This fact tells us also that ideal white noise cannot be observed in any case. The prove has been made on the line of ergodic theorem which is rewritten into the form which is easily handled.

It is often discussed how to choose the time interval of the measurement. This problem seems to be unimportant from the intuitive consideration, but was added for the completeness of this theory.

2. Ergodic Theorem

The definition of the autocorrelation function is given as:

$$\varphi(\tau) = \lim_{T=\infty} \frac{1}{2T} \int_{-T}^{T} f(t+\tau) f(t)\, dt . \tag{2.1}$$

The discrete form of (2.1) is

$$\varphi(\tau) = \lim_{N\to\infty} \frac{1}{N+1} \sum_{i=0}^{N} f(t_i + \tau) f(t_i) \tag{2.2}$$

where τ is the delay time.

When we choose the delay times successively, they fluctuate about the mean value τ_0, so that we have the observed value as follows:

$$\varphi^*(\tau_0) = \frac{1}{N+1} \sum_{i=0}^{N} f(t_i + \tau_0 + s_i) f(t_i) , \tag{2.3}$$