71. *Topology of Standard Path Spaces and Homotopy Theory.* I

By Hirosi TODA
Department of Mathematics, Osaka City University
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This is the first of a series of notes, whose aim is to clarify the homological structure of the path space $\Omega(X, A)=\{f: I^{1)}\to X \mid f(0)=*, f(1)\in A\}$ by means of "*standard path space*" and to investigate the homotopical structure of spaces. The paper of J-P. Serre[2] based on the singular homology theory of fibre spaces shows how the loop space $\Omega(X)=\Omega(X, *)$ is applied to the calculation of the Hurewicz homotopy groups $\pi_p(X)$ of X.

It was proved by J. B. Giever[3] that to every space X there exists a CW-complex $P(X)$ and a map of $P(X)$ into X inducing isomorphisms of the homotopy groups of $P(X)$ onto those of X. A problem to determine the homological structures of $P(\Omega(X))$ from those of $P(X)$ is closely related with Serre's theory. For the simply connected space X, this problem can be solved by selecting complexes $K(X)$ and $\omega(K(X))$, so-called a standard complex and a standard path complex respectively, when the complex $\omega(K(X))$ is combinatorially constructed from $K(X)$.

Here we give definitions of standard spaces and standard paths in them. The set of standard paths in a standard complex K, whose end points are in a subcomplex L of K, forms a closed subset $\omega(K, L)$ of $\Omega(K, L)$. The standard path space $\omega(K, L)$ is a CW-complex and is constructed from K and L by a combinatiorial method.

The fundamental result in this note is roughly stated as follows; *the injection: $\omega(K, L)\to\Omega(K, L)$ induces isomorphisms of homotopy and homology groups of $\omega(K, L)$ onto those of $\Omega(K, L)$.*

Our theory is applied to determine the orders of homotopy groups $\pi_p(S^n)$ of n-sphere S^n for $p\leqq n+8$.

§ 1. **Standard Paths in a Suspended Space.** Let $E(X)$ be a suspended space of a space X, which is obtained from $X\times I$ by shrinking a subset $*\times I\cup X\times \dot{I}^{4)}$ to a single point $*$, and let d: $X\times I\to E(X)$ be its shrinking map. Assume that a real function ρ of X is given such that ρ is positive excepting $\rho(*)=0$. Then define a standard path $l(x_1, \dots, x_n; y, t)$: $I\to E(X)$ by a formula

$$(A)\quad l(x_1, \dots, x_n;\ y, t)\ (s)=\begin{cases} d(x_i,\ (s-s_{i-1})/\rho(x_i)) & s_{i-1}\leqq s\leqq s_i, \\ d(y,\ (s-s_n)/\rho(y)) & s_n\leqq s\leqq 1, \end{cases}$$

where $x_i\in X$, $y\in A\subset x$, $t\in I$, $s_0=0$ and $s_i=\sum_{k=1}^{i}\rho(x_k)/(\sum_{k=1}^{n}\rho(x_k)+t\cdot\rho(y))$