70. *On Mixed Boundary Value Problems for a Circle*

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An explicit formula for the solution of a mixed boundary value problem in potential theory has recently been given by one of the present authors in the simplest case where the basic domain is the unit circle and there are two arcs on the circumference along which the values of the required function itself and of its normal derivative are prescribed[1]. The problem is formulated as follows: To determine a function $u(z)$ harmonic and bounded in the unit circle $|z|<1$ and satisfying the boundary conditions

$$u(e^{i\varphi})=U(\varphi) \text{ for } a<\varphi<b\,, \quad \frac{\partial u(e^{i\varphi})}{\partial\nu}=V(\varphi) \text{ for } b<\varphi<a+2\pi\,,$$

$\partial/\partial\nu$ denoting the differentiation along the inward normal at $e^{i\varphi}$.

The previous expression has been derived, as an illustration of the general discussion developed there, with the aid of a slit mapping function. In the present Note we shall first show that an equivalent formula can be derived in another way. Our present method of attack is based on a decomposition of the solution into two harmonic functions, of which the one solves a Dirichlet problem and the other a Neumann problem, and it applies efficiently also for the case where there are several arcs on the circumference of the unit circle along which the values of the function itself and of its normal derivative are alternately prescribed. The case of two pairs of boundary arcs will be explicitly treated in the latter part of the present Note.

Now, the solution $u(z)$ of the simplest problem formulated above may be regarded as the superposition of $u^{(1)}(z)$ and $u^{(2)}(z)$, i.e. $u(z)=u^{(1)}(z)+u^{(2)}(z)$, where $u^{(1)}(z)$ and $u^{(2)}(z)$ solve the problems with the special boundary conditions

$$u^{(1)}(e^{i\varphi}) = U(\varphi) \text{ and } \quad u^{(2)}(e^{i\varphi}) = 0 \quad \text{for } a<\varphi<b\,,$$
$$\frac{\partial u^{(1)}(e^{i\varphi})}{\partial\nu}=0 \quad \text{and} \quad \frac{\partial u^{(2)}(e^{i\varphi})}{\partial\nu}=V(\varphi) \quad \text{for } b<\varphi<a+2\pi\,.$$

Based on the special boundary character, the problems of deter-

1) Y. Komatu, Mixed boundary value problems. Journ. Fac. Sci. Univ. Tokyo **6** (1953), 345-391; cf. also a preparatory announcement made in Y. Komatu, Eine gemischte Randwertaufgabe für einen Kreis. Proc. Japan Acad. **28** (1952), 339-341. The general problem of this type has once discussed also by A. Signorini, Sopra un problema al contorno nella teoria delle funzioni di variabile complessa. Ann. Mat. Pura Appl. (3) **25** (1916), 253-273.