100. *On the Isomorphism of Topological Groups*

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In this note we shall prove some simple theorems on the identification of two topological groups with the same underlying abstract group.

1. Some notations and definitions. In the following we mean by the word "topology" a topology which satisfies Hausdorff's axioms.

We denote by $R(T)$ a set R with topology T (in this note R may be an abstract set or an abstract group or an abstract linear space). For two sets R_1 and R_2 (without topologies) we denote by $R_1 \times R_2$ their direct product, that is, the set of all pairs (x_1, x_2) where $x_1 \in R_1$ and $x_2 \in R_2$. When R_1 and R_2 are both abstract groups or both abstract linear spaces, we can consider $R_1 \times R_2$ as an abstract group or as an abstract linear space in the well-known manner (in the case of groups, we define $(x_1, x_2)(y_1, y_2)$ by (x_1y_1, x_2y_2) where $x_1, y_1 \in R_1$ and $x_2, y_2 \in R_2$, and in the case of linear spaces, we define $(x_1, x_2)+(y_1, y_2)$ and $\alpha(x_1, x_2)$ by (x_1+y_1, x_2+y_2) and $(\alpha x_1, \alpha x_2)$ respectively where $x_1, y_1 \in R_1$ $x_2, y_2 \in R_2$ and α is any real number). When $R_1(T_1)$ and $R_2(T_2)$ are two topological spaces, we denote by $R_1(T_1) \times R_2(T_2)$ the so-called topological direct product of $R_1(T_1)$ and $R_2(T_2)$. We denote the topology of the topological space $R_1(T_1) \times R_2(T_2)$ by $T_1 \times T_2$. Evidently by the definitions $R_1 \times R_2(T_1 \times T_2) = R_1(T_1) \times R_2(T_2)$. For a subset S of a topological space $R(T)$, we denote by $S\{T\}$ S with the topology induced by T. If R is endowed with two topologies T and T^*, and T is stronger (that is, with more open sets) than T^* or at least equivalent to T^*, then we write $T \geqq T^*$. By $\varDelta_R$, we denote the diagonal of $R \times R$, that is, the set of the elements of $R \times R$ which are of the form (a, a) where $a \in R$. When R is an abstract group or an abstract linear space, $\varDelta_R$ is a subgroup or a linear subspace of $R \times R$ respectively.

In the following, we shall say that a topological space is semi-compact, if it is locally bicompact and can be represented as a sum of a number, countable at most, of bicompact sets.

2. We prove first a simple lemma.

Lemma 1. *If R is endowed with three topologies T_1, T_2, T^* and $T_1 \geqq T^*$, $T_2 \geqq T^*$, then $\varDelta_R$ is closed in $R(T_1) \times R(T_2)$.*

Proof. $T_1 \times T_2 \geqq T^* \times T^*$, since $T_1 \geqq T^*$ and $T_2 \geqq T^*$. On the other hand, $\varDelta_R$ is closed in $R \times R(T^* \times T^*)$ $(=R(T^*) \times R(T^*))$, as