122. A Necessary Unitary Field Theory as a Non-Holonomic Parabolic Lie Geometry Realized in the Three-Dimensional Cartesian Space

By Tsurusaburo TAKASU

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The geometry based upon is the author's non-holonomic parabolic Lie geometry[3) *)], which is situated among other branches of geometry as follows: (Euclidean geometry): (Non-Euclidean geometry) = (parabolic Lie geometry): (Lie geometry) = (non-holonomic parabolic Lie geometry): (non-holonomic Lie geometry). Instead of the quadratic differential form:

(0.1) $$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = g_{\underline{\mu\nu}}dx^\mu dx^\nu + g_{\underset{\vee}{\mu\nu}}dx^\mu dx^\nu,$$

we take the linear vector form

(0.2) $$\gamma_5\omega^5 = \gamma_l\omega^l, \ (\omega^l = \omega^l_\mu dx^\mu, \ l = 1, 2, 3, 4),$$

such that

(0.3) $$dsds = \omega^5\omega^5 = \omega^l\omega^l,$$

where in Einstein's notation[1)] we have

(0.4) $$g_{\underline{\mu\nu}} = \omega^l_\mu\omega^l_\nu,$$

(0.5) $$g_{\underset{\vee}{\mu\nu}} = \gamma_4\gamma_1(\omega^4_\mu\omega^1_\nu - \omega^4_\nu\omega^1_\mu) + \cdots + \gamma_2\gamma_3(\omega^2_\mu\omega^3_\nu - \omega^3_\mu\omega^2_\nu)\cdots +,$$

and

(0.6) $$\gamma_1^2=\gamma_2^2=\gamma_3^2=-\gamma_4^2=\gamma_5^2=1, \ \gamma_4=i\gamma_5, \ \gamma_2\gamma_3+\gamma_3\gamma_2=0, \text{ etc.},$$
$$\gamma_4\gamma_1+\gamma_1\gamma_4=0, \text{ etc.}, \ \gamma_5\gamma_1+\gamma_1\gamma_5=0, \text{ etc.},$$

the $\gamma_1, \gamma_2, \gamma_3, \gamma_5$ being the Pauli's 4-4-matrices. Starting from (0.2) and pursuing necessities stepwise, the author will develop a unitary field theory.

1. *Realization of the Non-Holonomic Parabolic Lie Geometry in the Cartesian Space.* The said geometry will be realized in the three-dimensional Cartesian space provided with the Cartesian coordinates (ξ^i), $(i=1, 2, 3)$, such that

(1.1) $$d\xi^l = \omega^l,$$

(1.2) $$d\xi^4 = \omega^4 = dr,$$

the r being the radius of the oriented sphere with center $P(\xi^i)$. We adopt a double use for ds:

a vector (0.2) with components ω^l.	the common tangential segment $ds=idS$ of the oriented sphere (P, r) with its consecutive one.

The quantity $ds=idS$ is purely imaginary, when

*) The ciphers in the square brackets refer to the References attached to the end of this paper.