97. An Observation on the Brown-McCoy Radical

By F. SZÁSZ

Mathematical Institute of Academy of Science, Budapest

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We wish to characterize in this note the Brown-McCoy radical $G(A)$ of an associative ring A, as a radical $(1,1,1,1)(A)$, $(1,1,1,0)(A)$, $(1,1,0,1)(A)$ and $(1,2,1,1)(A)$, respectively, where $(k,l,m,n)(A)$ is a well-defined special F-radical of the ring A in the sense of Brown-McCoy [3] for arbitrary nonnegative integers k, l, m and n. The concept of a (k,l,m,n)-radicalring A can be illustrated by the following elementary remarks. If the elements of A form on the operation $a\circ b=a+b-ab\,(a,b\in A)$ a Neumann-regular semigroup (for instance in the case of a Jacobson-radicalring A, when $(A,0)$ is a group), then A is a $(k,0,1,1)$-radicalring and a $(0,l,1,1)$-radicalring at the same time for any integers $k,l\geqq 0$. Furthermore any (k,l,m,n)-semisimple ring A with minimum condition on *twosided* principal ideals is, as an (A,A)-doublemodule, completely reducible in a weak meaning, which generalizes the classical Wedderburn-Artin structure theorem also. (For the details of radicals, see [1], [2], [3].)

In this note the knowing of the results of Brown-McCoy [3] will be assumed for the reader. We denote the sum of all *twosided* principal ideals $(a^{(m)}\circ x\circ a^{(n)}-k\cdot a^{(l)})$ by $(k,l,m,n)(a)$, where a is a fixed element, X a varying element of A, $a\circ b=a+b-ab$, $a^{(0)}=0$, $a^{(1)}=a$, $a^{(k+1)}=a^{(k)}\circ a$ and k,l,m,n are nonnegative integers. An element $a\in A$ is called *(k,l,m,n)-regular*, if $a\in(k,l,m,n)(a)$. We call an element $a\in A$ *strictly (k,l,m,n)-regular*, if any element b of the *twosided* principal ideal (a) generated by a is *(k,l,m,n)-regular*. The set $(k,l,m,n)(A)$ of all strictly (k,l,m,n)-regular-elements of A is called the (k,l,m,n)-radical of A. *This is evidently a special F-radical of* A [3]. The rings with (k,l,m,n)-radical (0) are called (k,l,m,n)-semisimple. We call a subdirectly irreducible (k,l,m,n)-semisimple ring A shortly: (k,l,m,n)-primitive. An element $a\neq 0$ with the condition $(k,l,m,n)(a)=0$ is called here a (k,l,m,n)-distinguished element of A. By [3] the (k,l,m,n)-radical of A is the intersection of such ideals $\mathfrak{T}_\gamma\,(\gamma\in\Gamma)$ of A, that the factorrings $A/\mathfrak{T}_\gamma$ are (k,l,m,n)-primitive. $A/(k,l,m,n)(A)$ is (k,l,m,n)-semisimple, and a subdirect sum of (k,l,m,n)-primitive rings. By [3] a subdirectly irreducible ring A is (k,l,m,n)-primitive if and only if the minimal ideal $\mathfrak{D}\neq 0$ of A contains a (k,l,m,n)-distinguished element $d\neq 0$ playing the role of unity element in the case of radical