147. *On the Type of Completely Continuous Operators*

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1. Let A be an operator on a Hilbert space and let $\boldsymbol{R}(A)$ be a von Neumann algebra generated by A (i.e., the smallest von Neumann algebra containing A). Then A is said to be of type I (II, III) if $\boldsymbol{R}(A)$ is of type I (II, III). Clearly every normal operator A is of type I where $\boldsymbol{R}(A)$ is abelian. Moreover, every operator on a finite dimensional Hilbert space is of type I. Namely the classification described above has the essential meaning for non-normal operators on infinite dimensional Hilbert spaces. We shall concentrate our attention on the following question. Which non-normal operators are of type I? The answer is not much. In our recent paper [3] we have shown that an isometry is of the type I. This note is the second step in that direction.

That is, we shall prove the following theorem.

THEOREM. *A completely continuous operator on a Hilbert space is of type I.*

The class of completely continuous operators contains two important classes, the so-called Hilbert-Schmidt class and the trace class. Let A be an operator on a Hilbert space $\boldsymbol{H}$ and let $\{\varphi_i\}$ a family of complete orthonormal vectors in $\boldsymbol{H}$. Then the quantity $\sigma(A)=(\Sigma_i \|A\varphi_i\|^2)^{\frac{1}{2}}$ is independent of $\{\varphi_i\}$ and the operators A for $\sigma(A)<\infty$ form the Hilbert-Schmidt class. The product of two operators in the Hilbert-Schmidt class form the trace class. As is well known, every operator in the trace class is necessarily in the Hilbert-Schmidt class and every operator in the Hilbert-Schmidt class is necessarily completely continuous. Thus we shall obtain the following corollary.

COROLLARY. *An operator in the Hilbert-Schmidt class* (*or the trace class*) *is of type I.*

By an operator we shall mean a bounded linear transformation on a Hilbert space and for the terminology of von Neumann algebras we shall always refer to [1].

2. The first step is to decompose an arbitrary operator into type I, II and III components. If a von Neumann algebra $\boldsymbol{R}(A)$ generated by A is denoted by $\boldsymbol{M}$, it is easy to see that for each $E\in\boldsymbol{M}'$, a von Neumann algebra $\boldsymbol{M}_E$ which is the restriction of $\boldsymbol{M}$ to EH is generated by the restriction A_E of A to $\boldsymbol{EH}$. Thus, keeping in mind that there exists a unique family of mutually orthogonal