214. Fubini Theorems for Generalized Lebesgue-Bochner-Stieltjes Integral

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Let R be the space of reals. If Y_i, W $(i=1, \cdots, k)$ are seminormed spaces then by $L(Y_1, \cdots, Y_k; W)$ we shall denote the space of all operators u which are k-linear and continuous from the product of the spaces Y_i $(i=1, \cdots, k)$ into the space W. The seminorm of elements in the above spaces will be denoted by $|\ |$.

A family of sets V of an abstract space X will be called a pre-ring if for any two sets $A_1, A_2 \in V$ we have $A_1 \cap A_2 \in V$, and there exists disjoint sets $B_1, \cdots, B_k \in V$ such that $A_1 \backslash A_2 = B_1 \cup \cdots \cup B_k$.

A nonnegative function v on the pre-ring V will be called a volume if for every countable family of disjoint sets $A_t \in V$ $(t \in T)$ such that $A = \sum_T A_t \in V$ we have $v(A) = \sum_T v(A_t)$.

A triple (X, V, v) where V is a pre-ring of sets of X and v is a volume on V, will be called a volume space. If the triples (X_i, V_i, v_i) $(i=1, \cdots, k)$ are volume spaces then the triple (X, V, v) defined by $X = X_1 \times \cdots \times X_k$ and $V = V_1 \times \cdots \times V_k$ consisting of all sets of the form $A = A_1 \times \cdots \times A_k$; $A_i \in V_i$ with $v(A) = v_1(A_1) \cdots v_k(A_k)$ is a volume space.

Let (X, V, v) be a fixed volume space. Denote by $M_q(v, Z)$ $(1 < q \leq \infty)$ the space of all finite additive functions μ from the pre-ring V into a Banach space Z and such that $\mu(A) = 0$ if $v(A) = 0$ and

$$\sup \{(\textstyle\sum_n |\mu(A_n)|^q v(A_n)^{1-q})^{1/q}\} = ||\mu||_q < \infty$$

when $q \neq \infty$, where the supremum is taken over all finite families of disjoint sets $A_n \in V$ such that $v(A_n) > 0$. In the case when $q = \infty$ let $\sup \{|\mu(A)| v(A)^{-1} : A \in V\} = ||\mu||_q < \infty$ where the supremum is taken over all sets $A \in V$ such that $v(A) > 0$.

Now if $1/p_i + 1/q_i = 1$, $p_i \geq 1$, $i = 1, 2$ and $u \in L(Y_1, Y_2, Z; W)$, denote by $M(q_i, v_i, Z, u)$ the family of all functions $\mu(A_1, A_2)$ from $V_1 \times V_2$ into Z which are additive in each variable A_i separately and $\mu(A_1, A_2) = 0$ if $v_1(A_1) = 0$ or $v_2(A_2) = 0$; moreover assume that the following norm is finite $||\mu|| = \sup \{|\sum_{ij} u(y_{1i}, y_{2j}, \mu(A_{1i}, A_{2j}))(v_1(A_{1i}))^{-1/p_1}(v_2(A_{2j}))^{-1/p_2} a_{1i} a_{2j}|\}$ where the supremum is taken over all finite systems such that $||y_{1i}|| \leq 1$ $||y_{2j}|| \leq 1$, $\sum |a_{1i}|^{p_1} \leq 1$, $\sum |a_{2j}|^{p_2} \leq 1$, where A_{1i} is a family of disjoint sets of the pre-ring V_1 such that $v_1(A_{1i}) > 0$ and similarly A_{2j} is a finite family of disjoint sets of the pre-ring V_2 such that $v_2(A_{2j}) > 0$.