48. *Smooth Structures on $S^p \times S^q$*

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This paper shows the classification of smooth structures on $S^p \times S^q$ promised in [6].

In [10], Novikov classified smooth structures modulo one point of the manifolds which are tangentially homotopy equivalent to a product $S^p \times S^q$ of spheres. On the other hand, the author determined in his paper [6] the inertia group $I(S^p \times \tilde{S}^q)$ of $S^p \times \tilde{S}^q$. In the present paper, we shall show that combining these results derives complete classification of smooth structures on $S^p \times S^q$ for $p+q \geqq 6$, $1 \leqq p \leqq q$.

In the following we shall use the notations in [6].

Detailed proof will appear elsewhere.

1. Preliminaries. Let a smooth structure M_α on $S^p \times S^q$ be given i.e., assume that there is given a piecewise differentiable homeomorphism $f: S^p \times S^q \to M_\alpha$. Let x_0 (resp. y_0) denote a point of S^p (resp. S^q). Since $f(x_0 \times S^q)$ (resp. $f(S^p \times y_0)$) has a vector bundle neighbourhood in M_α, there exists a piecewise differentiable homeomorphism $h: M_\alpha \to M_\alpha$ such that $h(f(x_0 \times S^q))$ (resp. $h(f(S^p \times y_0))$) is a smooth submanifold of M_α (see R. Lashof and M. Rothenberg [9]). Therefore it follows that there exists a homotopy sphere $\tilde{S}^q$ (resp. $\tilde{S}^p$) which is embedded smoothly in M_α with a trivial normal bundle and which represents a generator of $H_q(M_\alpha) \cong H_q(S^p \times S^q) \cong Z$ (resp. $H_p(M_\alpha) \cong H_p(S^p \times S^q) \cong Z$) if $p \neq q$. We may assume that $\tilde{S}^p$ and $\tilde{S}^q$ intersect transversally at one point. Applying the similar argument as in [6], we can now show that

$$M_\alpha - \mathrm{Int}\, D^{p+q} = \tilde{S}^p \times D^q \veebar D^p \times \tilde{S}^q = \tilde{S}^p \times \tilde{S}^q - \mathrm{Int}\, D^{p+q}$$

where $\veebar$ denotes the plumbing of two manifolds. Hence M_α can be written as $\tilde{S}^p \times \tilde{S}^q \# \tilde{S}^{p+q}$ for some exotic sphere $\tilde{S}^{p+q}$, here $\#$ denotes the connected sum. It is easily seen that this still holds in the case $p=q$. Obviously $\tilde{S}^p \times \tilde{S}^q \# \tilde{S}^{p+q}$ is tangentially homotopy equivalent to $S^p \times S^q$. Therefore, by making use of the classification theorem of Novikov [10], we see that $\tilde{S}^p \times \tilde{S}^q$ is diffeomorphic to $S^p \times \tilde{S}^q$ modulo one point for $p \leqq q$. Thus the problem of classifying smooth structures on $S^p \times S^q$ $(p \leqq q)$ is reduced to the study of smooth structures of the form $S^p \times \tilde{S}^q \# \tilde{S}^{p+q}$.

2. Lemmas. The following lemma is proved in Theorem C of [6].