109. *A Non-Commutative Integration Theory for a Semi-Finite AW*-algebra and a Problem of Feldman*

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We shall extend Feldman's result on "Embedding of AW^*-algebras" to semi-finite AW^*-algebras, that is, we shall show that a semi-finite AW^*-algebra with a separating set of states which are completely additive on projections (c.a. states) has a faithful representation as a semi-finite von Neumann algebra. Full proofs will appear elsewhere.

Let M be a semi-finite AW^*-algebra with a separating set $\mathfrak{S}$ of c.a. states. By a c.a. state ϕ on M we mean a state on M such that for any orthogonal family of projections $\{e_i\}$ in M with $e=\sum_i e_i$ $\phi(e)=\sum_i \phi(e_i)$. Let C be the algebra of "measurable operators" affiliated with M [6]. Denote the set of all positive elements, projections, partial isometries and unitary elements in M by M^+, M_p, M_{pi} and M_u, respectively.

Let $\tilde{\mathfrak{S}}$ be the set of finite linear combinations of elements in $\{a^*\omega a, \omega \in \mathfrak{S}, a \in M\}$, where $(a^*\omega a)(x)=\omega(axa^*)$ for all $x \in M$. For any positive number ε and any positive integer n, put $V_{\varepsilon,n}(\omega_1, \omega_2, \cdots, \omega_n)(0) =\{a ; |\omega_i(a)|<\varepsilon, i=1,2,\cdots n, \omega_1, w_2, \cdots, \omega_n \in \tilde{\mathfrak{S}}\}$ and we define the $\sigma(\mathfrak{S})$-topology of M by assigning sets of the form $V_{\varepsilon,n}(\omega_1, \omega_2, \cdots, \omega_n)(0)$ to be its neighborhood system of 0. Since $\tilde{\mathfrak{S}}$ is a separating set of continuous linear functionals on M, this topology is the separated locally convex topology defined by the family of semi-norms $q_\omega(x)=|\omega(x)|$, $\omega \in \tilde{\mathfrak{S}}$. Then we have, by [3, Lemma 3],

Lemma 1. *Let $\{e_\alpha\}\alpha \in A$ be an orthogonal set of projections in M such that $e=\mathrm{Sup}\ [\sum\{e_\alpha, \alpha \in I\}$, $A \supset I \in F(A)$ where $F(A)$ is the family of all finite subsets of A], then $\sum\{e_\alpha, \alpha \in I\} \rightarrow e(I \in F(A))$ in the $\sigma(\mathfrak{S})$-topology.*

Lemma 2. *Any abelian AW^*-subalgebra, especially, the center Z of M is a W^*-algebra* ([7]) *and the $\sigma(\mathfrak{S})$-topology restricted to this subalgebra is equivalent to the σ-topology on bounded spheres.*

Let $\boldsymbol{Z}$ be the set of all $[0, +\infty]$-valued continuous functions on the spectrum of Z [1], then we have

Theorem 1. *There is an operation Φ from M^+ to $\boldsymbol{Z}$ having the following properties:*

(i) $\Phi(h_1+h_2)=\Phi(h_1)+\Phi(h_2)$ $h_1, h_2 \in M^+$;

(ii) $\Phi(\lambda h)=\lambda\, \Phi(h)$ *if λ is a positive number and $h \in M^+$;*