140. *The Structure of Quasi-Minimal Sets*

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1. Introduction. The concept of the quasi-minimal sets, introduced by H. F. Hilmy [1], plays rather important roles for the investigation of the structure of the center of the compact dynamical systems.

In this paper, we study mainly the three problems, i.e., (a) how a quasi-minimal set contains minimal sets, (b) the qualities of these minimal sets, (c) the behaviors of the orbits near these minimal sets. Main results obtained are as follows:

Theorems 9 and 10 for (a),

Theorems 8, 12 and 13 for (b), and

Theorem 14 for (c).

2. Definitions and notations.

X: a compact metric space.

R: a real line.

π: $X\times R\to X$ is a mapping which satisfies

1) $\pi\in C[X\times R]$,

2) $\pi(x, 0)=x$, and

3) $\pi(\pi(x, s), t)=\pi(x, s+t)$.

The triple (X, R, π) is a compact dynamical system whose phase space, phase group, and phase projection are X, R, and π, respectively.

$\gamma(x)=\{\pi(x, t)\,;\, t\in R\}$ is the orbit passing through $x\in X$.

$\gamma^+(x)=\{\pi(x, t)\,;\, t\geqq 0\}$ and $\gamma^-(x)=\{\pi(x, t)\,;\, t\leqq 0\}$ are respectively positive semi-orbit and negative semi-orbit from $x\in X$.

$\Lambda^+(x)=\bigcap_{0\leqq t}\overline{\gamma^+(\pi(x, t))}$ and $\Lambda^-(x)=\bigcap_{0\geqq t}\overline{\gamma^-(\pi(x, t))}$ are the positive and negative limit set of $\gamma(x)$, respectively.

$\gamma(x)$ is positively (negatively) Poisson stable if and only if $\Lambda^+(x)\cap\gamma(x)\neq\phi$ $(\Lambda^-(x)\cap\gamma(x)\neq\phi)$.

$\gamma(x)$ is Poisson stable if and only if it is both positively and negatively Poisson stable.

$\gamma(x)$ is positively (negatively) asymptotic if and only if $\gamma(x)\cap\Lambda^+(x)=\phi$ and $\Lambda^+(x)\neq\phi$ $(\gamma(x)\cap\Lambda^-(x)=\phi$ and $\Lambda^-(x)\neq\phi)$.

A subset S of X is invariant if and only if $\gamma(x)\subset S$ holds for any $x\in S$.

A closed and invariant set F is minimal if and only if it contains no proper subsets which are closed and invariant.