242. *On Products of Quasi-Perfect Maps*

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1. Introduction. The purpose of this paper is to give a necessary and sufficient condition for the product map to be a quasi-perfect map under the condition that the image of the product map is a sequential space. Namely we shall prove

Theorem 1. *Let* $f_i: X_i \to Y_i$ $(i=1,2,\cdots)$ *be quasi-perfect maps and* $\prod_{i=1}^{\infty} Y_i$ *a sequential space. Then the following properties are equivalent.*

(a) $\prod_{i=1}^{\infty} f_i$ *is a quasi-perfect map.*

(b) $\prod_{i=1}^{\infty} X_i$ *satisfies the condition* (C_1) *below;*

(C_1): *If each* K_i *is a closed countably compact subset of* X_i, *then* $\prod_{i=1}^{\infty} K_i$ *is countably compact in* $\prod_{i=1}^{\infty} X_i$.

(c) $\prod_{i=1}^{\infty} f_i^{-1}(C_i)$ *is countably compact in* $\prod_{i=1}^{\infty} X_i$ *for each convergent sequence* C_i *in* Y_i. *Here, by a convergent sequence we mean the union of the sequence and its limit point.*

According to S. P. Franklin [4], a space X is called a sequential space if a subset F of X is closed whenever $F \cap C$ is closed for every compact metric subset C of X, and such a space is precisely the quotient of a metric space.

First countable spaces are sequential. Of course, sequential spaces are k-spaces. But the converse does not hold. Indeed, the Stone-Cěch compactification of a normal and non-compact space is not sequential.

We assume all maps are continuous and onto, and all spaces are T_2.

2. Proof of Theorem 1. First of all, we consider the following condition (C_0) on a space X.

(C_0): Each sequence whose closure is countably compact has a subsequence whose closure is compact.

The condition (k_0) in T. Chiba [2] implies the condition (C_0), and the converse is true in regular q-spaces in the sense of E. Michael [7].

A space X is called isocompact by P. Bacon [1] if every closed countably compact is compact.

Lemma 1. *K-spaces, locally isocompact spaces, sequentially compact spaces, and regular spaces whose each point is a* G_δ*-set all satisfy the condition* (C_0).

Proof. Let X be a k-space and $\{x_i; i \in N\}$ a sequence in X whose closure is countably compact. We assume all points x_i are distinct.