194. *The Completion of Topological Spaces*

By Suketaka MITANI
University of Osaka Prefecture

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For a set X, the family consisting of all the filters in X is denoted by $F(X)$. For a topological space X, the collection of all the open sets of X is called *a topology* of X and denoted by $O(X)$.

Let X be a topological space. If a filter $\mathfrak{f}$ in X is generated by the filter base $\mathfrak{f}\cap O(X)$, then $\mathfrak{f}$ is an *open filter* in X. And the family consisting of all the open filters in X is denoted by $OF(X)$. Specially, for a point x of X, the open filter generated by the filter base $\{V\mid x\in V\in O(X)\}$ is called a *neighborhood system* of x and denoted by $\mathfrak{N}(x)$.

If a topological space X contains its dense subspace Y, then X is said to be an *extension* of Y.

Let a topological space X be an extension of Y. Then, for a point x of X and its neighborhood system $\mathfrak{N}(x)$, $\{V\cap Y\mid V\in\mathfrak{N}(x)\}$ is a *trace* of x on Y. We get a mapping φ of X into $OF(Y)$ such that, for every $x\in X$, $\varphi(x)$ is the trace of x on Y. This φ is called a *trace system* of X on Y. And the restriction $\varphi\mid X\backslash Y$ of φ on $X\backslash Y$ is a *tracer* of X on Y.

If φ is a trace system of an extension X of a topological space Y on Y, then X is said to be *extended* from Y by a tracer $\varphi\mid X\backslash Y$.

The following is the fundamental theorem of the extension theory of topological spaces.

Theorem 1. *Let Y be a topological space, X be a set containing Y and φ be a mapping of $X\backslash Y$ into $OF(X)$. Then there exists a topology of X such that X is an extension of which the tracer on Y is φ.*

In this paper, instead of this Theorem 1, Theorem 2 will be proved.

Example 1. Let X be the discrete topological space consisting of all the natural numbers, X^* be $X\cup\{\omega_1,\omega_2\}$, $\mathfrak{N}(\omega_1)$ be $\{A\cup\{\omega_1\}\mid A\subseteq X, X\backslash A \text{ is finite}\}$ and $\mathfrak{N}(\omega_2)$ be $\{A\cup\{\omega_2\}\mid A\subseteq X, X\backslash A \text{ is finite}\}$. Then X^* is a T_1 extension of X.

Example 2. Let X be the same as Example 1, X^* be $X\cup\{\omega_1,\omega_2\}$, $\mathfrak{N}(\omega_1)$ be $\{A\cup\{\omega_1,\omega_2\}\mid A\subseteq X, X\backslash A \text{ is finite}\}$ and $\mathfrak{N}(\omega_2)=\mathfrak{N}(\omega_1)$. Then X^* is an extension of X.

Example 3. Let R be the topological space of all the real numbers and S be the subspace of R consisting of all the rational numbers. Denote a trace of a real number x on S by $\varphi(x)$. If x is a rational