185. δ_p and Countably Paracompact Spaces

By J. M. BOYTE
Appalachian State University

(Comm. by Kinjirô KUNUGI, M. J. A., May 12, 1971)

In [3], Mack defines the term δ-normal, and proves that if I is the closed unit interval, then a space X is countably paracompact if and only if $X\times I$ is δ-normal. In this paper we define the term δ_p which is stronger than δ-normal, but is strictly weaker than countable paracompactness, and is strictly weaker than normality; and we prove the following:

Theorem 1. *The following are equivalent for a space X*

(i) *The space X is countably paracompact.*

(ii) *The space X is δ_p and countably metacompact.*

(iii) *The space X is δ_p and every countable open cover of X has a countable semi-refinement of closed sets.*

(iv) *If C is a countable open cover of X, then there exists a countable collection $L=\{L_i \mid i=1,2,\cdots\}$ of open refinements of C such that for each $x\in X$ there is some L_i that is locally finite with respect to x.*

(v) *If I is the closed unit interval, then $X\times I$ is δ_p.*

We observe that (ii) of the above theorem is a slight generalization of a condition proven by Dowker [2]; further; we point out that Theorem 1 in [3] is used in proving (v) of the above theorem.

Definition. If X is a space and C is an open cover of X, then L is a semi-refinement of C if each member of L is contained in the union of a finite subset of C.

Definition. If X is a space and L is a collection of subsets of X, then L is locally finite with respect to a subset A of X, if for each $x\in A$, there exists an open set V, $x\in V$, such that V intersects only finitely many members of L.

Definition. Let X be a space and let N be a cardinal number. Then X is called an N_p space, if for each open cover C, cardinality of C less than or equal N, there exists for each closed set F contained in any member of C, an open refinement of C that is locally finite with respect to F. In the special case when $N=$aleph zero, we will denote N_p by δ_p.

For an infinite cardinal N, a topological space is N-normal if each pair of disjoint closed sets, one of which is a regular G_N-set, have disjoint neighborhoods [3]. A set B is called a regular G_N-set if it is the intersection of at most N closed sets whose interiors contain B [3].