228. Markov Semigroups with Simplest Interaction. II

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We have defined the semigroup with simplest interaction in Part I. In this Part II, we give the definition of the Markov processes with simplest interaction, their decompositions and constructions, and finally our main result, the relation to the branching Markov processes.

Q is always assumed to be a compact Hausdorff space with a countable basis. We employ the notation of Part I.

§1. Definition.

1.0. Let $\bar{X}=(\bar{P}_x, \bar{X}_t)$ be a Markov process with state space $Q_* \cup \{\Delta\}$ whose transition semigroup $(\bar{T}_t)_{t\geqq 0}$ maps $C_0(Q_*)$ into itself and is strongly continuous. Here $Q_* \cup \{\Delta\}$ is the one point compactification of the locally compact space Q_* and, for any element ϕ in $C_0(Q_*)$, we set $\phi(\Delta)=0$.

1.1. Definition. The process $\bar{X}$ is called a *Markov process with simplest interaction* (or briefly, *process with interaction*) if its transition semigroup $(\bar{T}_t)_{t\geqq 0}$ is a semigroup with simplest interaction on $C_0(Q_*)$.

The process $\bar{X}$ is assumed in this paper to be a Hunt process. Since the non-interaction part $(\bar{T}_t^0)_{t\geqq 0}$ of $(\bar{T}_t)_{t\geqq 0}$ constructed in §3 of Part I is dominated by the latter, it is a transition semigroup of some subprocess $(\bar{P}_x, \bar{X}_t^0)$ of $(\bar{P}_x, \bar{X}_t)$; in fact, setting

$$(1)\qquad R(\omega)=\inf\{t: \bar{X}_t(\omega)\notin Q_n\} \qquad \text{if } \bar{X}_0(\omega)\in Q_n,\ n\geqq 1,$$

it follows from Theorem 3.1. of Part I that

$$(2)\qquad \bar{T}_t^0\phi(\bar{x})=\bar{E}_x[\phi(\bar{X}_t)1_{\{t<R\}}] \qquad (t\geqq 0,\ \bar{x}\in Q_*)$$

for any $\phi\in C_0(Q_*)$ where 1_A is the indicator function of the set A.

1.2. Definition. The Markov time R is called *first interacting time*. The Markov process $(\bar{P}_X, \bar{X}_t^0)$ is called *non-interacting part*.

Let $X=(P_x, X_t)$ be the Markov process obtained by piecing together the process $\bar{X}^0$. We suppose that X is conservative, which is possible if we assume that the constant functions belong to the domain $\mathcal{D}(\bar{A}^0)$ of the infinitesimal generator $\bar{A}^0$ of $(\bar{T}_t^0)_{t\geqq 0}$ in the Hille-Yosida sense. Since $(\bar{T}_t^0)$ is degenerated, so is the transition semigroup $(T_t)_{t\geqq 0}$ of the process X. It is easy to verify that X is equivalent to an n independent copies of some Hunt process $x=(P_x, x_t)$ with state space Q if $X_0\in Q_n$ and $n\geqq 1$.

1.3. Definition. The process x is called *base process* of the process with interaction X. The space Q is referred to the *base space*.