227. *Remark on the Essential Spectrum of Symmetrizable Operators*

By Yoshio ITAGAKI
Department of Mathematics, Miyagi University of Education, Sendai

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1. Introduction. We will use the following notion and notations. We mainly refer to [3]; see also [1].

Let T be a closed linear operator in a Banach space E, $\rho(T)$ its resolvent set, and $\sigma(T)$ its spectrum. The dimension of the null space of T, $N(T)$, written $\alpha(T)$, will be called the *kernel index* of T and the *deficiency* of the range T in E, $R(T)$, written $\beta(T)$, will be called the *deficiency index* of T. The *index* $\kappa(T)$ is defined by

$$\kappa(T)=\alpha(T)-\beta(T).$$

If the operator T has a finite index, it is called a *Fredholm operator.*

We denote by $\sigma_{em}(T)$ the set of all complex number λ for which $T-\lambda I$ is not a Fredholm operator with index zero and call it the *essential spectrum* of T. The set of points of $\sigma(T)$ which is not an isolated eigenvalue λ of finite multiplicity, namely $\alpha(T-\lambda I)<\infty$, will be denoted by $\sigma_0(T)$. Here an isolated eigenvalue means an eigenvalue which is an isolated point of the spectrum.

Let X be a Banach space and H a Hilbert space such that

i) *$X\subset H$, and the embedding mapping; $X\to H$ is continuous,*

ii) *X is dense in H.*

The purpose of this paper is to prove the following theorem:

Theorem. *Let T be a closed linear operator in X and essentially self-adjoint in H, that is, its smallest closed extension (or its closure) in H is self-adjoint. Then*

$$\sigma_0(T\,|\,X)=\sigma_{em}(T\,|\,X).$$

Here we denoted by $T\,|\,X$ the operator considered in X. Similarly we will denote by $\bar{T}$ the closure of T in H, $\sigma(\bar{T}\,|\,H)$ the spectrum of $\bar{T}$ in H and so on.

Since the index of the Fredholm operator is invariant under the addition of compact operators [3, Theorem V.2.1], in particular, when K is a linear compact operator in X,

$$\sigma_{em}(T\,|\,X)=\sigma_{em}(T+K\,|\,X).$$

In addition, if K is symmetrizable, that is, symmetric with respect to the inner product of H, $T+K$ is essentially self-adjoint [4, p. 288, Theorem 4.4] and, by our Theorem