223. *On an Extension Theorem for Turning Point Problem*

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§1. Introduction. Let us consider the asymptotic properties of solutions of differential equations depending on a small positive parameter ε when ε tends to zero. If there are some properties which do not stand uniformly for the region considered that is, there exist points in the region which we call singular points where the asymptotic nature breaks down, the studies of such phenomenon are often called the singular perturbation problems. Many important problems from the applied mathematics and theoretical physics can be converted to the research of these problems. The boundary layer theory of fluid mechanics, the relaxation oscillation in the circuit theory, and the turning point problems in quantum mechanics are familiar among others.

In this paper, we restrict ourselves to consider the asymptotic expansions of solutions of the second order ordinary differential equations containing turning points. At the turning point, an asymptotic expansion in power series of ε breaks down, and if we want to have uniformly valid asymptotic expansion in the region containing turning points, it is necessary to consider much more complicated series of ε than integral power series of ε.

Here we talk about the stretching and matching method used frequently in fluid mechanics. This consists of the following three procedures, at first an asymptotic expansion of solution which is conveniently called outer solution is calculated in some region where no singular point contains, nextly an asymptotic expansion called inner solution is obtained in a small neighborhood of singular point by appropriate stretching transformations of independent variable, and thereafter the relation between the outer solution and the inner solution is considered. To perform these procedures rigorously, the two regions of existence of the outer and inner solutions must overlap with each other for all sufficiently small ε. For this purpose the following Kaplun's Extension theorem is an essential tool and its proof is given, for example, in Eckhaus [2].

Kaplun's extension theorem.

(i) *Let* $F(x,\varepsilon)$ *be defined for* (x,ε) *in* $[0<x\leqq x_0, 0<\varepsilon\leqq\varepsilon_0]$ *and* $\lim_{\varepsilon\to 0} F(x,\varepsilon)=0$, uniformly in $x_1\leqq x\leqq x_0$ for all $x_1>0$.