# 211. *A Perturbation Theorem for Linear Contraction Semigroups on Reflexive Banach Spaces*

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**1. Introduction and statement of the result.** This paper is concerned with the perturbation of linear contraction semigroups on Banach spaces. Our result gives a partial extension of a perturbation theorem for such semigroups obtained by Nelson [5] and Gustafson [1].

A linear operator $A$ (with domain $D(A)$ and range $R(A)$) in an arbitrary Banach space $X$ is said to be *accretive* if

(A) $\|(A+\xi)u\| \geqq \xi \|u\|$ for every $u \in D(A)$ and $\xi > 0$.

This implies that $(A+\xi)^{-1}$ exists and $\|(A+\xi)^{-1}v\| \leqq \xi^{-1}\|v\|$, $v \in R(A+\xi)$, for $\xi > 0$. It can be shown that $(A+\xi)^{-1}$ has domain $X$ either for every $\xi > 0$ or for no $\xi > 0$; in the former case we say that $A$ is *m-accretive*. Also, (A) is equivalent to the following condition:

(A′) For every $u \in D(A)$ there is $f \in F(u)$ such that

$$\mathrm{Re}\,(Au, f) \geqq 0,$$

where $F$ denotes the duality mapping: $F(u) = \{f \in X^*; (u, f) = \|u\|^2 = \|f\|^2\}$ (cf. Kato [3] in which the term "monotonic" was used instead of "accretive"). Note that the inequality is not required to hold for every $f \in F(u)$. But if $X$ is *reflexive* and $A$ is $m$-accretive, then the inequality holds for every $f \in F(u)$. This is a consequence of the following facts (cf. Lumer-Phillips [4], Remark 1 to Theorem 3.1):

1) $-A$ is the (infinitesimal) generator of a linear contraction semigroup on an arbitrary Banach space if and only if $A$ is $m$-accretive and densely defined;

2) If $X$ is reflexive, then $m$-accretive operators in $X$ are necessarily densely defined (cf. Kato [2], or Yosida [9], p. 218).

In fact, being the generator of a linear contraction semigroup is independent of the multiplicity of duality mapping.

On the (relatively bounded) perturbation of linear contraction semigroups, we know the following result due to Nelson and Gustafson (cf. [1]):

**Theorem 1.** *Let $A$ and $B$ be linear operators in an arbitrary Banach space $X$ such that*

$$\|Bu\| \leqq a\|u\| + b\|Au\|, \quad a \geqq 0, \quad 0 < b < 1, \quad u \in D(A) \subset D(B).$$

If $-A$ is the generator of a linear contraction semigroup and if $B$ is