# 203. *On the Stability of Solutions of Some Non-Autonomous Differential Equations of the Third Order*

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**1. Introduction.** In this note we investigate the asymptotic stability in the large, as $t\to\infty$, of the zero solution of the differential equation

(1.1) $$\dddot{x}+\psi(t, x, \dot{x}, \ddot{x})+\phi(t, x, \dot{x})+c(t)f(x)=0,$$

where $\psi$, $\phi$, $f$ and $c$ are real valued functions. The dots indicate differentiation with respect to $t$ and all solutions considered are assumed real.

In [4] K. E. Swick established conditions under which all solutions of the non-autonomous equations

(1.2) $$\dddot{x}+p(t)\ddot{x}+q(t)g(\dot{x})+r(t)h(x)=0$$

(1.3) $$\dddot{x}+f(t, x, \dot{x})\ddot{x}+q(t)g(\dot{x})+r(t)h(x)=0$$

tend to the zero solution as $t\to\infty$.

Recently, in [2] T. Hara also obtained some conditions under which all solutions of the equation

(1.4) $$\dddot{x}+a(t)\ddot{x}+b(t)\dot{x}+c(t)x=0$$

(1.5) $$\dddot{x}+a(t)f(x, \dot{x})\ddot{x}+b(t)g(x, \dot{x})\dot{x}+c(t)x=0$$

tend to the zero solution as $t\to\infty$.

To prove the following theorems (see 3) we construct a Liapunov function using the well-known techniques which are frequent in [3].

**2. Auxiliary Lemmas.** Consider the sytem of differential equation

(2.1) $$\dot{X}=F(t, X)$$

where $X=(x_1, \cdots, x_n)$, $F(t, 0)=0$ for $t\in I=[0, +\infty)$ and $F(t, X)$ is continuous in $I\times R^n$.

The following lemmas are well-known and play the essential role to prove the theorems, see [5] (Th. 8.1, Th. 10.2, Th. 14.2).

**Lemma 2.1.** *Suppose that there exists a Liapunov function $V(t, X)$ defined on $0\leqq t<\infty$, $\|X\|<H(H>0)$, which satisfies the following conditions*; (i) $V(t, 0)\equiv 0$, (ii) $w_1(\|X\|)\leqq V(t, X)$, *where $w_1(r)$ is a continuous increasing, positive definite function,* (iii) $\dot{V}_{(2.1)}(t, X)\leqq 0$. *Then, the zero solution $X(t)\equiv 0$ of the system* (2.1) *is stable.*

**Lemma 2.2.** *Suppose that there exists a Liapunov function $V(t, X)$*