202. On the Asymptotic Behavior of Solutions of Certain Third Order Ordinary Differential Equations

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(Comm. by Kenjiro SHODA, M. J. A., Sept. 13, 1971)

1. Introduction. Our purpose here is to study the behavior as $t\to\infty$ of solutions of the differential equations

(1.1) $$\dddot{x}+a(t)\ddot{x}+b(t)\dot{x}+c(t)x=e(t) \qquad \left(\dot{x}=\frac{dx}{dt}\right),$$

(1.2) $$\dddot{x}+a(t)\ddot{x}+b(t)\dot{x}+c(t)h(x)=e(t),$$

(1.3) $$\dddot{x}+a(t)f(x,\dot{x})\ddot{x}+b(t)g(x,\dot{x})\dot{x}+c(t)h(x)=e(t).$$

We assume the following conditions throughout this note.

(c_1) $a(t)$, $b(t)$ and $c(t)$ are positive and continuously differentiable functions on $[0,\infty)$.

(c_2) $e(t)$ is continuous and absolutely integrable on $[0,\infty)$.

(c_3) $h(x)$ is continuously differentiable and real-valued for all x.

(c_4) $f(x,y)$, $f_x(x,y)$, $g(x,y)$ and $g_x(x,y)$ are continuous and real-valued for all (x,y).

In [2], the author considered the conditions under which all solutions of the non-autonomous equations (1.1) and (1.3) with $e(t)\equiv 0$ and $h(x)=x$ tend to zero as $t\to\infty$.

2. Theorems.

Theorem 1. *Suppose that $a(t)$, $b(t)$ and $c(t)$ are continuously differentiable and $e(t)$ is continuous on $[0,\infty)$ and following conditions are satisfied;*

(i) $A\geqq a(t)\geqq a_0>0$, $B\geqq b(t)\geqq b_0>0$, $C\geqq c(t)\geqq c_0>0$ *for* $t\in[0,\infty)$,

(ii) $xh(x)>0$ $(x\neq 0)$, $H(x)=\int_0^x h(\xi)d\xi\to+\infty$ *as* $|x|\to\infty$,

(iii) $\dfrac{a_0b_0}{C}>h_1\geqq h'(x)$,

(iv) $\mu a'(t)+b'(t)-\dfrac{1}{\rho}c'(t)<\dfrac{a_0b_0-Ch_1}{2}$ $\qquad \left(\mu=\dfrac{a_0b_0+Ch_1}{2b_0},\ \rho=\dfrac{\mu}{h_1}\right)$,

(v) $\int_0^\infty |c'(t)|\,dt<\infty$, $c'(t)\to 0$ *as* $t\to\infty$,

(vi) $\int_0^\infty |e(t)|\,dt<\infty$.

Then every solution $x(t)$ of (1.2) *is uniform-bounded and satisfies $x(t)\to 0$, $\dot{x}(t)\to 0$, $\ddot{x}(t)\to 0$ as $t\to\infty$.*

Corollary 1. *Suppose that the conditions* (i), (v), (vi) *and in addi-*