130. *On Pseudoparacompactness and Continuous Mappings*

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Throughout this paper we assume that spaces are completely regular T_1-spaces and maps are continuous. The completion of a space X with respect to its finest uniformity is called the topological completion of X, and denoted by μX. According to Morita [8] a space X is called pseudoparacompact (resp. pseudo-Lindelöf) if μX is paracompact (resp. Lindelöf).

As for these notions, in the same paper Morita proved the following remarkable results.

Theorem 1 (Morita [8], Theorems 3.1, 3.2 and 3.5).

(1) *μX is compact iff X is pseudocompact.*

(2) *μX is always a paracompact M-space for any M-space X.*

(3) *Let X be an M-space. X is pseudo-Lindelöf iff it is the quasi-perfect inverse image of a separable metric space.*

The characterizations of pseudoparacompactness and pseudo-Lindelöfness have been obtained by Howes [4] and Ishii [5] independently. On the other hand, in [2] Hanai and Okuyama (cf. Isiwata [6]) essentially proved the following result: "*If a space X is the inverse image of a pseudocompact space under an open quasi-perfect map, then X is pseudocompact*". Here the assumption that the map is open cannot be dropped in general ([3] Example 2.4). Analogously to this result, in § 1 we shall prove the following theorem which is a partial answer to a problem posed by Ishii [5] concerning (2) and (3) of Theorem 1: "*Is pseudoparacompactness or pseudo-Lindelöfness preserved under taking the inverse image by a quasi-perfect* (*or perfect*) *map?*"

Theorem 2. *If there is an open quasi-perfect map $\varphi: X \to Y$ from a space X onto a pseudoparacompact* (*resp. pseudo-Lindelöf*) *space Y, then X is pseudoparacompact* (*resp. pseudo-Lindelöf*).

In § 2, by virtue of recent results obtained by Morita, we shall prove the following

Theorem 3. *Let $\varphi: X \to Y$ be an open quasi-perfect map from a space X onto a space Y.*

(1) *If μY is locally compact and paracompact, then so is μX.*

(2) *If μY is σ-compact, then so is μX.*

§ 1. Proof of Theorem 2. Before proving Theorem 2, we shall