10. Probabilities on Inheritance in Consanguineous Families. III

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III. Simple mother-descendants combinations (Continuation)

3. General mother-descendants combination

The problems in the preceding sections concern a combination consisting of an individual and its two collateral descendants in which a collateral separation takes place at the original generation. We shall now consider a mother-descendants combination in which a collateral separation appears at a certain intermediate generation. In fact, we introduce a probability

$$\pi_{\iota|\mu\nu}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2) \equiv \bar{A}_{\alpha\beta}\kappa_{\iota|\mu\nu}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)$$

which is defined by an equation

$$\kappa_{\iota|\mu\nu}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2) = \sum \kappa_\iota(\alpha\beta;\ ab)\kappa_{\mu\nu}(ab;\ \xi_1\eta_1,\ \xi_2\eta_2).$$

According to three systems for the $\kappa_{\mu\nu}$'s, we distinguish here also *three systems*, i.e. $\mu=\nu=1$, $\mu=1<\nu$ or $\mu>1=\nu$, and $\mu,\nu>1$.

The formula for the lowest system is then expressed in the form

$$\kappa_{\iota|11}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2) = \sigma(\xi_1\eta_1,\ \xi_2\eta_2)+2^{-\iota+1}U(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2),$$

where the quantity U is defined by

$$U(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2) = \sum Q(\alpha\beta;\ ab)\kappa(ab;\ \xi_1\eta_1,\ \xi_2\eta_2).$$

It is symmetric with respect to $\xi_1\eta_1$ and $\xi_2\eta_2$, and its values are listed as follows; cf. a remark stated at the end of I, § 1:

$U(ii;\ ii,\ ii) = \frac{1}{8}i(1-i)(1+i)(1+2i)$, $U(ii;\ ii,\ ig) = \frac{1}{4}ig(1-2i^2)$,
$U(ii;\ ii,\ gg) = \frac{1}{8}ig^2(1-2i)$, $U(ii;\ ii,\ fg) = \frac{1}{4}ifg(1-2i)$,
$U(ii;\ ik,\ ik) = \frac{1}{8}k(1+k-3i^2+ik-8i^2k)$,
$U(ii;\ ik,\ kk) = \frac{1}{8}k^2(1-3i+k-4ik)$,
$U(ii;\ ik,\ ig) = \frac{1}{8}kg(1+i-8i^2)$, $U(ii;\ ik,\ kg) = \frac{1}{8}kg(1-3i+2k-8ik)$,
$U(ii;\ ik,\ gg) = \frac{1}{8}kg^2(1-4i)$, $U(ii;\ ik,\ fg) = \frac{1}{4}kfg(1-4i)$,
$U(ii;\ kk,\ kk) = -\frac{1}{8}k^2(1+k)(1+2k)$, $U(ii;\ kk,\ kg) = -\frac{1}{8}k^2g(3+4k)$,
$U(ii;\ kk,\ gg) = -\frac{1}{4}k^2g^2$, $U(ii;\ kk,\ fg) = -\frac{1}{2}k^2fg$,
$U(ii;\ hk,\ hk) = -\frac{1}{8}hk(2+3h+3k+8hk)$, $U(ii;\ hk,\ kg) = -\frac{1}{8}hkg(3+8k)$,
$U(ii;\ hk,\ fg) = -hkfg$;
$U(ij;\ ii,\ ii) = \frac{1}{16}i(1-2i)(1+i)(1+2i)$,
$U(ij;\ ii,\ ij) = \frac{1}{16}i(i+2j+i^2-3ij-8i^2j)$,
$U(ij;\ ii,\ jj) = \frac{1}{16}ij(i+j-4ij)$, $U(ij;\ ii,\ ig) = \frac{1}{16}ig(2-3i-8i^2)$,
$U(ij;\ ii,\ jg) = \frac{1}{16}ig(i+2j-8ij)$, $U(ij;\ ii,\ gg) = \frac{1}{16}ig^2(1-4i)$,
$U(ij;\ ii,\ fg) = \frac{1}{8}ifg(1-4i)$,