1. *Theory of Path Structure.* I

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1. **Introduction.** The present note is a preliminary report of the paper which will be published elsewhere under the same title.

Veblen[1] constructed a descriptive geometry in terms of "point" and "order" undefined, and on the other hand Prenowitz[2] formulated descriptive geometries as multigroups. Our aim is to show, making use of the Prenowitz's algebraic idea, that the way found by Veblen leads up directly to the fertile land of geometries of manifolds. The geometry of manifold which we are going to construct may be considered as an abstraction of the geometry of paths which was also inaugulated by Veblen with Eisenhart.[3] But a larger part of the studies included in the present paper is concerned to so-called flat manifolds. We shall expect that sooner or later our theory will extend to more general cases.

2. **Concept of Path Structure.** Suppose that there is a non-empty set S whose elements are called *points*. We denote points by small Latin letters, and sets of points (subsets of S) will be designated by capital Latin letters. We write $A\approx B$ if $A \cap B \neq 0$.

We now assume that there is an operation $-$, which associates a certain set of points $a-b$ called the *difference* of a and b with each ordered pair of points a, b. The set union $\bigcup\{a-b \mid a\approx A,\ b\approx B\}$ for non-empty A, B is denoted $A-B$; we define $A-0=0-A=0$ for an arbitrary A. The set of all x for which $x-b\approx a$ is designated by $a+b$, and it is called the *sum* of a and b. If A, $B\neq 0$, then $A+B$ is the set union $\bigcup\{a+b \mid a\approx A,\ b\approx B\}$. We define further $A+0=0+A=0$ for an arbitrary A.

The set S thus furnished with the operation $-$ will be denoted $(S\,;\,-)$. We assume that $(S\,;\,-)$ satisfies the followings:

I. *Idempotent Law*: $a-a=a$.

II. *Commutative Law*: $a+b=b+a$.

III. *Absorptive Law*: $a+(a-b)\subset a-b$.

IV. *Reductive Law*: *If* $a+b\approx a+c$ *and* $b\neq c$, *then we have either* $b\approx a+c$, $b\neq c$ *or* $c\approx a+b$, $b\neq c$.

V. *If* $a-b\neq 0$, *then* $a+b\neq 0$.

Then $(S\,;\,-)$ is referred to as a *path structure*. From now on our investigations will be carried out in the path structure $(S\,;\,-)$.

A directed pair (a, b) is called *subtractive* if $a-b\neq 0$. A