32. Probabilities on Inheritance in Consanguineous Families. IV

By Yûsaku KOMATU and Han NISHIMIYA
Department of Mathematics, Tokyo Institute of Technology
(Comm. by T. FURUHATA, M.J.A., Feb. 12, 1954)

IV. Ancestors-descendant combinations through an intermediate marriage

1. Ancestor-parent-child combination immediate after a marriage

Suppose that two individuals $A_{\alpha\beta}$ and $A_{\gamma\delta}$ are accompanied by their μth and νth descendants A_{ab} and A_{cd}, respectively, and that these descendants are married and originate themselves an nth descendant $A_{\xi\eta}$. Let the probability of a triple consisting of $(A_{\alpha\beta}, A_{\gamma\delta}; A_{\xi\eta})$ be then designated by

$$\overline{A}_{\alpha\beta}\overline{A}_{\gamma\delta}\varepsilon_{\mu\nu;n}(\alpha\beta, \gamma\delta; \xi\eta).$$

The probability of parents-descendant combination, ε_n, treated in I, § 2, may be regarded to correspond to the *lowest case* $\mu=\nu=0$; in particular, $\varepsilon_1\equiv\varepsilon_{00;1}$ represents nothing but ε. Here we distinguish *four systems* in case of higher generation-numbers μ, ν according to $\mu>0=\nu$, $n=1$ or $\mu=0<\nu$, $n=1$; $\mu>0=\nu$, $n>1$ or $\mu=0<\nu$, $n>1$; $\mu>0$, $\nu>0$, $n=1$; and $\mu>0$, $\nu>0$, $n>1$.

The *first system* will now be treated. By virtue of an evident quasi-symmetry property with respect to $\alpha\beta$, $\gamma\delta$ and μ, ν, it suffices to consider the former. Its defining equation

$$\varepsilon_{\mu0;1}(\alpha\beta, \gamma\delta; \xi\eta)=\sum\kappa_\mu(\alpha\beta; ab)\varepsilon(ab, \gamma\delta; \xi\eta)$$

can be brought into the form

$$\varepsilon_{\mu0;1}(\alpha\beta, \gamma\delta; \xi\eta)=\kappa(\gamma\delta; \xi\eta)+2^{-\mu}C_0(\alpha\beta, \gamma\delta; \xi\eta),$$

where C_0 is defined by

$$C_0(\alpha\beta, \gamma\delta; \xi\eta)=2\sum Q(\alpha\beta; ab)\varepsilon(ab, \gamma\delta; \xi\eta).$$

The values of C_0 are set out as follows:

$C_0(ii, ii; ii)=1-i$, $\quad C_0(ii, ii; ig)=-g$;
$C_0(ii, ik; ii)=\frac{1}{2}(1-i)$, $\quad C_0(ii, ik; ik)=\frac{1}{2}(1-i-k)$,
$C_0(ii, ik; kk)=-\frac{1}{2}k$, $\quad C_0(ii, ik; ig)=-\frac{1}{2}g$,
$C_0(ii, ik; kg)=-\frac{1}{2}g$;
$C_0(ii, kk; ik)=1-i$, $\quad C_0(ii, kk; kk)=-k$,
$C_0(ii, kk; kg)=-g$;
$C_0(ii, hk; ik)=\frac{1}{2}(1-i)$, $\quad C_0(ii, hk; hk)=-\frac{1}{2}(h+k)$;
$C_0(ij, ii; ii)=\frac{1}{2}(1-2i)$, $\quad C_0(ij, ii; ij)=\frac{1}{2}(1-2j)$,

*) I-III, Proc. Japan Acad. **30** (1954), 42-52.