52. *Probabilities on Inheritance in Consanguineous Families. VIII*

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(Comm. by T. FURUHATA, M.J.A., March 12, 1954)

VIII. Mother-descendants combinations through several consanguineous marriages (Continuation)

3. General mother-descendants combinations through several consanguineous marriages

In the present section we consider the problems which correspond to those discussed in VI, § 3, but we now suppose that there exist two descendants instead of one. The reduced probability in consideration is then defined by

$$\kappa_{l\,|\,(\mu\nu;\,n)_t\,|\,\mu\nu}(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2)=\sum\kappa_{l\,|\,(\mu\nu;\,n)_t}(\alpha\beta;\,ab)\kappa_{\mu\nu}(ab;\,\xi_1\eta_1,\,\xi_2\eta_2)\quad(\mu\nu=\mu_{t+1}\nu_{t+1}).$$

In case $\mu=\nu=1$, we get the following results:

$$\kappa_{l\,|\,(\mu\nu;\,1)_t\,|\,11}(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2)=\sigma(\xi_1\eta_1,\,\xi_2\eta_2)+4(u_t+w_t)\mathfrak{X}(\xi_1\eta_1,\,\xi_2\eta_2)$$
$$+2^{-l+1}\{2^{-t}\Lambda_tU(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2)+(v_t+2w_t)\mathfrak{Y}(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2)\},$$

$$\kappa_{l\,|\,(\mu\nu;\,n)_t\,|\,11}(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2)=\sigma(\xi_1\eta_1,\,\xi_2\eta_2)+2^{-l-N_t+1}\Lambda_tU(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2)\quad\text{for } n_t>1.$$

In case $\mu=1<\nu$ we get the following results:

$$\kappa_{l\,|\,(\mu\nu;\,1)_t\,|\,1\nu}(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2)=\sigma_{1\nu}(\xi_1\eta_1,\,\xi_2\eta_2)+2^{-\nu+1}(u_t+w_t)\overline{A}_{\xi_1\eta_1}Q(\xi_1\eta_1;\,\xi_2\eta_2)$$
$$+2^{-l-t}\Lambda_t\{\overline{A}_{\xi_2\eta_2}Q(\alpha\beta;\,\xi_1\eta_1)+2^{-\nu+1}V(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2)\}$$
$$+2^{-l-\nu}(v_t+2w_t)S(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2),$$

$$\kappa_{l\,|\,(\mu\nu;\,n)_t\,|\,1\nu}(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2)=\sigma_{1\nu}(\xi_1\eta_1,\,\xi_2\eta_2)$$
$$+2^{-l-N_t}\Lambda_t\{\overline{A}_{\xi_2\eta_2}Q(\alpha\beta;\,\xi_1\eta_1)+2^{-\nu+1}V(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2)\}\quad\text{for } n_t>1.$$

In case $\mu,\ \nu>1$, we get the following results:

$$\kappa_{l\,|\,(\mu\nu;\,1)_t\,|\,\mu\nu}(\alpha\beta;\,\xi_1\eta_1,\xi_2\eta_2)=\sigma_{\mu\nu}(\xi_1\eta_1,\xi_2\eta_2)+2^{-\lambda+1}(u_t+w_t)\overline{A}_{\xi_1\eta_1}Q(\xi_1\eta_1;\,\xi_2\eta_2)$$
$$+2^{-l-t+1}\Lambda_t\{2^{-\mu}\overline{A}_{\xi_2\eta_2}Q(\alpha\beta;\,\xi_1\eta_1)+2^{-\nu}\overline{A}_{\xi_1\eta_1}Q(\alpha\beta;\,\xi_2\eta_2)\}$$
$$+2^{-l-\lambda}(2^{-t+1}\Lambda_t+v_t+2w_t)S(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2),$$

$$\kappa_{l\,(\mu\nu;\,n)_t\,\mu\nu}(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2)=\sigma_{\mu\nu}(\xi_1\eta_1,\,\xi_2\eta_2)$$
$$+2^{-l-N_t+1}\Lambda_t\{2^{-\mu}\overline{A}_{\xi_2\eta_2}Q(\alpha\beta;\,\xi_1\eta_1)+2^{-\nu}\overline{A}_{\xi_1\eta_1}Q(\alpha\beta;\,\xi_2\eta_2)$$
$$+2^{-\lambda}S(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2)\}\quad\text{for } n_t>1.$$

More generally, we obtain the following results:

$$\kappa_{l\,|\,(\mu\nu;\,n)_t\,|\,(\mu'\nu';\,1)_{t'}\,|\,11}(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2)=\sigma(\xi_1\eta_1,\,\xi_2\eta_2)+4(u'_{t'}+w'_{t'})\mathfrak{X}(\xi_1\eta_1,\,\xi_2\eta_2)$$
$$+2^{-l-N_t+1}\Lambda_t\{2^{-t'}\Lambda'_{t'}U(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2)+(v'_{t'}+2w'_{t'})S(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2)\},$$

$$\kappa_{l\,(\mu\nu;\,n)_t\,|\,(\mu'\nu';\,1)_{t'}\,|\,1\nu'}(\alpha\beta;\,\xi_1\eta_1,\,\xi_2\eta_2)$$
$$=\sigma_{1\nu'}(\xi_1\eta_1,\,\xi_2\eta_2)+2^{-\nu'+1}(u'_{t'}+w'_{t'})\overline{A}_{\xi_1\eta_1}Q(\xi_1\eta_1;\,\xi_2\eta_2)$$