50. *Probabilities on Inheritance in Consanguineous Families. VI*

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VI. Mother-descendant combinations through several consanguineous marriages

1. Special combination with several consanguineous marriages

We have discussed in the preceding chapter[1] the mother-descendant combinations through a single consanguineous marriage. We now attempt to generalize the problem to the case where several consanguineous marriages intervene between a mother and her descendant. Our present purpose is to determine the probability of a combination which consists of the original individual $A_{\alpha\beta}$ and of its descendant $A_{\xi\eta}$, consanguineous marriages interjacent between them occurring t times, and which is designated, with an easily comprehensible routine notation, by

$$\pi_{\mu_1\nu_1;n_1|\cdots|\mu_t\nu_t;n_t}(\alpha\beta;\,\xi\eta)\equiv\bar{A}_{\alpha\beta}\kappa_{\mu_1\nu_1;n_1|\cdots|\mu_t\nu_t;n_t}(\alpha\beta;\,\xi\eta)$$

or briefly, provided no confusion can arise, by

$$\pi_{(\mu\nu;n)_t}(\alpha\beta;\,\xi\eta)\equiv\bar{A}_{\alpha\beta}\kappa_{(\mu\nu;n)_t}(\alpha\beta;\,\xi\eta).$$

By definition, the reduced probability $\kappa_{(\mu\nu;n)_t}$ is given by

$$\kappa_{(\mu\nu;n)_t}(\alpha\beta;\,\xi\eta)=\sum\prod_{r=1}^{t}\kappa_{\mu_r\nu_r;n_r}(a_{r-1}b_{r-1};\,a_rb_r)$$

with $a_0b_0\equiv\alpha\beta$ and $a_tb_t\equiv\xi\eta$, where the summation extends over all the possible sets of $t-1$ genotypes $A_{a_rb_r}$ $(r=1,\ldots,t-1)$.

We can really permit here also the degenerate cases where some of the n_r's with $1\leqq r<t$ vanish. But, these exceptional cases will be postponed to a later chapter. In the present section we shall deal with a special case where the n's are all equal to unity. Its defining equation then becomes

$$\kappa_{(\mu\nu;1)_t}(\alpha\beta;\,\xi\eta)=\sum\kappa_{(\mu\nu;1)_{t-1}}(\alpha\beta;\,ab)\kappa_{\mu_t\nu_t;1}(ab;\,\xi\eta),$$

whence follows, as verified by induction, the formula

$$\kappa_{(\mu\nu;1)_t}(\alpha\beta;\,\xi\eta)=\bar{A}_{\xi\eta}+2^{-t+1}\Lambda_tQ(\alpha\beta;\,\xi\eta)+4u_tR(\xi\eta)$$
$$+2v_tS(\alpha\beta;\,\xi\eta)+2w_tT(\alpha\beta;\,\xi\eta);$$

$$\Lambda_t=\prod_{r=1}^{t}L_r\equiv\prod_{r=1}^{t}(2^{-\mu_r}+2^{-\nu_r}),\qquad u_t=\sum_{r=1}^{t-1}\prod_{s=r+1}^{t}2^{-\lambda_s-2},$$

$$v_t=\sum_{r=1}^{t-1}2^{-r+1}\Lambda_r\prod_{s=r+1}^{t}2^{-\lambda_s-2},\qquad w_t=\prod_{s=1}^{t}2^{-\lambda_s-2};$$

1) Cf. Proc. Japan Acad. **30** (1954), 152–155. There the value of $S(ij;\,ik)$ (p. 154, l. 6) should be read $-\frac{1}{4}k(1-4i)$ instead of $-\frac{1}{4}k(1-4k)$.