57. On the Integration of the Temporally Inhomogeneous Diffusion Equation in a Riemannian Space. II

By Kôsaku YOSIDA
Department of Mathematics, Osaka University
(Comm. by K. KUNUGI, M.J.A., April 12, 1954)

1. **Introduction.** In a preceding note with the same title,[1] the author devised an existence proof of the solution for the Cauchy's problem of the temporally inhomogeneous diffusion equation with C^∞ coefficients:

$$(1.1)\qquad \frac{\partial f(t,s,x)}{\partial t} - A_{tx} f(t,s,x) = 0, \quad t>s,$$

$$\lim_{t\to s} f(t,s,x) = f(x) \in L_1(R)^{2)} \text{ almost everywhere,}$$

$$A_{tx} f(t,s,x) = g(x)^{-1/2} \frac{\partial^2}{\partial x^i \partial x^j}(g(x)^{1/2} a^{ij}(t,x) f(t,s,x))$$

$$- g(x)^{-1/2} \frac{\partial}{\partial x^i}(g(x)^{1/2} b^i(t,x) f(t,s,x)) + c(t,x) f(t,s,x),$$

in a connected domain R of an m-dimensional, orientable C^∞ Riemannian space with the metric $ds^2 = g_{ij}(x) dx^i dx^j$.

The purpose of the present note is to show that the existence proof in [I] may be modified so as to yield the existence proof of the solution admitting the kernel representation

$$(1.2)\qquad f(t,s,x) = \int_R P(t,s,x,y) f(y) dy \quad \text{for every } f(x) \in L_1(R).$$

2. **The Proof of the Kernel Representation.** Let D denote a linear set of C^∞ functions with compact carriers such that D is $L_1(R)$-dense in $L_1(R)$. We regard $A_t = A_{tx}$ as an additive operator on $D \subseteqq L_1(R)$ to $L_1(R)$, and let $\overline{A}_t$ be the smallest closed extension of A_t. We assume that D is so chosen that the following Hypothesis is satisfied.

Hypothe is: Let, for sufficiently large integer n (independently of t), the resolvents

$$(2.1)\qquad I_t^{(n)} = (I - n^{-1}\overline{A}_t)^{-1}$$

1) Proc. Japan Acad., **30** (1954), No. 1, 19-23. This note will be referred to as [I]. At this juncture, the author wants to give the following corrigenda to [I]: (i) On page 23, lines 9–10, "the second term" and "the third term" should be read respectively as "the third term" and "the fourth term". (ii) On page 23, line 24, "is equivallent to" should be read as "equivalent, when $x \in V(x_0)$, to".

2) The Banach space of Borel measurable functions which are integrable, with respect to the measure $dx = g(x)^{1/2} dx^1 \dots dx^m$, over R.