70. On the Generation of a Strongly Ergodic Semi-Group of Operators

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1. Introduction. A fundamental problem of a semi-group of bounded linear operators $T(\xi)$, $0 < \xi < \infty$, from a complex Banach space X into itself is to characterize the infinitesimal generator which determines the structure of a semi-group of operators.

Such a problem has been discussed by E. Hille [1][1] and K. Yosida [2] for a semi-group of operators satisfying the following conditions:

(c_1) $T(\xi)$ *is strongly continuous at zero,*

(c_2) $\|T(\xi)\| \leq 1 + \beta\xi$ *for sufficiently small* ξ,

where β is a constant. Later their results were generalized to a semi-group of operators satisfying only the condition (c_1) by R. S. Phillips [3] and the present author [4]. Further this result has been generalized to a strongly measurable semi-group of operators by W. Feller [5].

In this paper we shall deal with the above problem concerning a semi-group of operators which is strongly Abel (or Cesàro) ergodic to the identity at zero.[2] We sketch here our results. The details will appear in the Tôhoku Mathematical Journal.

2. Semi-group of operators strongly Abel ergodic at zero
Let $\{T(\xi);\ 0 < \xi < \infty\}$ be a semi-group of operators satisfying the following conditions:

(a) *For each* $\xi > 0$, $T(\xi)$ *is a bounded linear operator from a complex Banach space* X *into itself and*

$$T(\xi + \eta) = T(\xi) \cdot T(\eta) = T(\eta) \cdot T(\xi).$$

(b) $T(\xi)$ *is strongly measurable in* $(0, \infty)$.

We may further assume the following condition without loss of generality:

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1) Numbers in brackets refer to the references at the end of this paper.

2) After this paper was written up, the author found the abstract of Phillips' paper [6], in which he writes that the necessary and sufficient conditions that a closed linear operator be the c.i.g. (the smallest closed extension of the infinitesimal generator) of a semi-group of operators which is strongly Abel (or Cesàro) ergodic (summable) to the identity at zero are obtained, but the detail is not obvious for the present author.