# 95. On Hannerisation of Two Countably Paracompact Normal Spaces

By Kiyoshi ISÉKI
Kobe University

(Comm. by K. KUNUGI, M.J.A., June 12, 1954)

In this note, we shall prove the following

*Theorem 1. The Hannerisation of two countably paracompact normal spaces is countably paracompact normal.*

A space $X$ is called *countably paracompact*, if every countable open covering of $X$ has a locally finite open refinement. For normal space, such a space $X$ can be characterized by the following condition: *every countable open covering of $X$ has a star finite open refinement.* For the proof, see K. Iséki (3).

Let $X$ and $Y$ be normal spaces, $B$ a closed subset of $Y$ and $f:B\rightarrow X$ a mapping (continuous). Let $X\cup Y$ be the free union of $X$ and $Y$, and $Z$ the identification space obtained from $X\cup Y$ by identifying $x\in B$ with $f(x)\in X$. The natural mapping of $X\cup Y$ onto $Z$ induces two mappings $j:X\rightarrow Z$ and $k:Y\rightarrow Z$. That is to say a subset $O$ of $Z$ is open if, and only if, $j^{-1}(O)$ and $k^{-1}(O)$ are open. Such a $Z$ is called the Hannerisation of $X$ and $Y$. It is well known that $X$ is closed in $Z$ and the partial mapping $k/Y-B$ is a homeomorphism onto $Z-X$.

O. Hanner [(1), (2)] proved that, if $X$ and $Y$ are both normal (resp. collectionwise normal, paracompact), then so is $Z$. E. Michael (5) observed that a similar result for perfectly normal space holds true. The present author (4) proved that, if $X$ and $Y$ are completely normal spaces, then so is $Z$.

*Proof of Theorem 1.* It is clear that $Z$ is normal. Let $\alpha=\{O_n\}$ be any countable open covering of $Z$, then we shall show that $\alpha$ has a locally finite open refinement. The open covering $\{O_n\cap X\}$ of $X$ has a star finite open refinement $\{U_n\}$, since $X$ is countably paracompact normal. We can take $O_{i_n}$ such that $U_n\subset O_{i_n}$ for each $U_n$. By a theorem of O. Hanner [(2), Lemma 7.2], there is a locally finite open covering $\{W_n\}$ of $Z$ such that $U_n=W_n\cap X$. We can suppose that $W_n\subset O_{i_n}$ replacing $W_n$ by $W_n\cap O_{i_n}$. If $Z=\bigcup_{n=1}^{\infty}W_n$, $Z$ is countably paracompact, and if it is not, Hanner method [(2), p. 330] is available for our proof. Let $W=\bigcup_{n=1}^{\infty}W_n$, then $W$ is an open set in $Z$ such that $W\supset X$. Thus $k^{-1}(W)$ is open in $Y$, and $k^{-1}(W)\supset Z$. By the normality of $Y$, there is an open set $V$ in $Y$