92. A Proof for a Theorem of M. Nakaoka

By Katuhiko MIZUNO
Osaka City University, Osaka

(Comm. by K. KUNUGI, M.J.A., June 12, 1954)

1. Let X be a simply connected topological space with vanishing homotopy groups $\pi_i(X)$ for $i<n$, $n<i<q$ and $q<i$. Then M. Nakaoka[1] proved that the transgression τ in the Cartan-Serre fiber space associated with X and the geometrical realization $\bar{\boldsymbol{k}}_n^{q+1}$ of the Eilenberg-MacLane invariant $\boldsymbol{k}_n^{q+1}$ are related as follows:

(1) $$\tau\boldsymbol{b}=-\bar{\boldsymbol{k}}_n^{q+1},$$

where $\boldsymbol{b}$ is the basic cohomology class of the fiber.

The purpose of this note is to construct a singular structure of an arbitrary fiber space (E, p, B) satisfying

(2) (i) the total space E is a simply connected space with vanishing homotopy groups $\pi_i(E)$ for $i>q$ with a base point e_0,

(ii) the base space B is a space with vanishing homotopy groups $\pi_i(B)$ for $i\geqq q$ with a base point $b_0=p(e_0)$,

(iii) the projection $p: E\longrightarrow B$ induces the isomorphisms $\pi_i(E)\approx\pi_i(B)$ for $i<q$,

(iv) the fiber $F=p^{-1}(b_0)$ is a space with a base point e_0.

And, as an application, we shall give a proof of the similar relation as (1) in an arbitrary fiber space satisfying (2) about the Postnikov invariant.[2]

This paper makes full use of the results and terminologies of the preceding paper by the author.[3]

2. Let Y be a topological space. A singular n-simplex T of Y is a function $T(x_0, \dots, x_n)\in Y$ defined for $0\leq x_i, x_0+x_1+\cdots+x_n=1$. For any element $\beta=\sum_j m_j\beta_j$ of $K_r(n)$, the β-face T_β of T is an r-chain defined as

$$T_\beta=\sum_j m_j T_{\beta_j}, \quad T_{\beta_j}(x_0, \dots, x_r)=T(y_0, \dots, y_n),$$

where $y_i=0$ if $i\neq\beta_j(k)$ for all $k=0, \dots, r$, and $y_i=\sum_k x_k$ for $\beta_j(k)=i$. In particular, the ε^i-face of T will be denoted simply by $T^{(i)}$ and is called the i-th face.

---

1) M. Nakaoka: Transgression and the invariant $\boldsymbol{k}_n^{q+1}$, Proc. Japan Acad., **30**, 363–368 (1954).

2) Refer 3). Originally reported in the Math. Reviews, **13** (1952). (M. M. Postnikov: Doklady Akad. Nauk URSS., **76**, 359–362 (1951); ibid., **76**, 789–791 (1951)).

3) K. Mizuno: On the minimal complexes, Jour. Inst. Polytech., Osaka City Univ., **5**, 41–51 (1954).