91. *Note on Topological Transitivity*

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M. Morse and G. A. Hedlund solved the problem of the topological transitivity for each two-dimensional closed orientable Riemannian manifold $\sum$ of class C^3 and of genus $p>1$ provided that no geodesic on $\sum$ has on it two mutually conjugate points [4]. I have shown the one method of symbolic representation already [1]. In this paper we shall show the new proof of topological transitivity as an application of the symbolic representation. (Cf. Morse-Hedlund [2],[3].)

1. We already know the following theorems of symbolic representation.

Theorem 1. If there be given any regular geodesic relative to P on $\sum$, there exists one, and only one unending regular sequence whose generating symbols are $\tilde{a}_i, \tilde{b}_i, \tilde{a}_i^{-1}, \tilde{b}_i^{-1}$.

Theorem 2. If there be given any unending regular sequence whose generating symbols are $\tilde{a}_i, \tilde{b}_i, \tilde{a}_i^{-1}, \tilde{b}_i^{-1}$, there exists at least one geodesic which corresponds to the given regular sequence.

Now we prepare some definitions.

Definition 1. Any geodesic or geodesic ray on $\sum$ is represented by a curve on phase space Ω of $\sum$. If its closure coincides with Ω, we say that the geodesic or geodesic ray is transitive.

Definition 2. Any symbolic ray will be termed transitive if it contains a copy of all regular subblocks.

2. Lemma 1. There exists a transitive regular symbolic ray.

Proof. As the set of regular blocks is enumerable, we denote them $A_1, A_2, A_3, \ldots$.
Then the ray

$$X=A_1e_1A_2e_2A_3e_3\ldots$$

is regular if the symbols e_i are successively chosen so as to satisfy the conditions (1) and (2) of regular sequence. (Cf. [1].) It is evident that X is transitive.

Theorem 3. In the case $p>1$ if the non-conjugacy hypothesis holds good, two geodesic rays with the same initial point on Φ can not be of the same type.

Proof. Let two geodesic rays r_1, r_2 with the same initial point on Φ be of the same type and f be the mapping explained in my