136. *Probabilities on Inheritance in Consanguineous Families.* X

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VIII. Combinations through the most extreme consanguineous marriages

4. Parent-descendants combinations

We shall now deal with *parent-descendants combinations of the extreme mode*. We begin with a combination *immediate after successive consanguineous marriages*, of which the probability is given by

$$\mathfrak{k}_{t-1|11}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)\equiv\kappa_{(11;0)_{t-1}|11}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)$$
$$=\sum\kappa(\alpha\beta;\ ab,\ cd)\mathfrak{e}_{t-1}(ab,\ cd;\ \xi_1\eta_1,\ \xi_2\eta_2)$$

or alternatively by

$$\mathfrak{k}_{t-1|11}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)=\sum\overline{A}_{ab}\mathfrak{e}_t(ab,\ \alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2).$$

Actual computation will lead to the following results:

$$\mathfrak{k}_{t-1|11}(ii;\ ii,\ ii)=\frac{1}{2}(1+i)+(1-i)\Big\{-\boldsymbol{R}\frac{7+3\sqrt{5}}{10}\omega^t+\frac{1}{5}\frac{1}{4^t}+i\Big(-\boldsymbol{R}\frac{2+\sqrt{5}}{10}\omega^t-\frac{1}{2}\frac{1}{2^t}$$
$$-\frac{3}{10}\frac{1}{4^t}\Big)\Big\},$$

$$\mathfrak{k}_{t-1|11}(ii;\ ii,\ ig)=g\Big\{\boldsymbol{R}\frac{2}{5}\omega^t-\frac{2}{5}\frac{1}{4^t}+i\Big(\boldsymbol{R}\frac{-1+\sqrt{5}}{10}\omega^t+\frac{1}{2}\frac{1}{2^t}+\frac{3}{5}\frac{1}{4^t}\Big)\Big\},$$

$$\mathfrak{k}_{t-1|11}(ii;\ ii,\ gg)=g\Big\{\boldsymbol{R}\frac{3-\sqrt{5}}{10}\omega^t-\frac{1}{30}\frac{1}{2^t}+\frac{1}{30}\frac{1}{4^t}-\frac{1}{6}\frac{1}{(-4)^t}-\frac{2}{15}\frac{1}{(-8)^t}+i\Big(\boldsymbol{R}\frac{-2+\sqrt{5}}{10}\omega^t$$
$$+\frac{1}{30}\frac{1}{2^t}-\frac{2}{15}\frac{1}{4^t}+\frac{1}{6}\frac{1}{(-4)^t}+\frac{2}{15}\frac{1}{(-8)^t}\Big)+g\Big(\frac{1}{30}\frac{1}{2^t}+\frac{1}{6}\frac{1}{4^t}+\frac{1}{6}\frac{1}{(-4)^t}+\frac{2}{15}\frac{1}{(-8)^t}\Big)\Big\},$$

$$\mathfrak{k}_{t-1|11}(ii;\ ii,\ fg)=fg\Big(\frac{1}{15}\frac{1}{2^t}+\frac{1}{3}\frac{1}{4^t}+\frac{1}{3}\frac{1}{(-4)^t}+\frac{4}{15}\frac{1}{(-8)^t}\Big),$$

$$\mathfrak{k}_{t-1|11}(ii;\ ik,\ ik)=k\Big\{\boldsymbol{R}\frac{-4+4\sqrt{5}}{5}\omega^t-\frac{4}{15}\frac{1}{2^t}+\frac{2}{15}\frac{1}{4^t}+\frac{2}{3}\frac{1}{(-4)^t}+\frac{4}{15}\frac{1}{(-8)^t}+i\Big(\boldsymbol{R}\frac{6-2\sqrt{5}}{5}\omega^t$$
$$+\frac{4}{15}\frac{1}{2^t}-\frac{8}{15}\frac{1}{4^t}-\frac{2}{3}\frac{1}{(-4)^t}-\frac{4}{15}\frac{1}{(-8)^t}\Big)+k\Big(\frac{4}{15}\frac{1}{2^t}+\frac{2}{3}\frac{1}{4^t}-\frac{2}{3}\frac{1}{(-4)^t}+\frac{4}{15}\frac{1}{(-8)^t}\Big)\Big\},$$

$$\mathfrak{k}_{t-1|11}(ii;\ ik,\ kk)=k\Big\{\boldsymbol{R}\frac{2}{5}\omega^t-\frac{1}{3}\frac{1}{2^t}+\frac{1}{10}\frac{1}{4^t}-\frac{1}{6}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{-1+\sqrt{5}}{10}\omega^t-\frac{1}{6}\frac{1}{2^t}+\frac{1}{10}\frac{1}{4^t}$$
$$+\frac{1}{6}\frac{1}{(-4)^t}\Big)+k\Big(\frac{1}{3}\frac{1}{2^t}-\frac{1}{2}\frac{1}{4^t}+\frac{1}{6}\frac{1}{(-4)^t}\Big)\Big\},$$

$$\mathfrak{k}_{t-1|11}(ii;\ ik,\ ig)=kg\Big(\frac{4}{15}\frac{1}{2^t}+\frac{2}{3}\frac{1}{4^t}-\frac{2}{3}\frac{1}{(-4)^t}-\frac{4}{15}\frac{1}{(-8)^t}\Big),$$

$$\mathfrak{k}_{t-1|11}(ii;\ ik,\ kg)=kg\Big(\frac{4}{15}\frac{1}{2^t}-\frac{1}{3}\frac{1}{4^t}+\frac{1}{3}\frac{1}{(-4)^t}-\frac{4}{15}\frac{1}{(-8)^t}\Big),$$

$$\mathfrak{k}_{t-1|11}(ii;\ ik,\ gg)=kg\Big(\frac{1}{15}\frac{1}{2^t}-\frac{1}{6}\frac{1}{4^t}-\frac{1}{6}\frac{1}{(-4)^t}+\frac{4}{15}\frac{1}{(-8)^t}\Big),$$

$$\mathfrak{k}_{t-1|11}(ii;\ kk,\ kk)=\frac{1}{2}k+k\Big\{-\boldsymbol{R}\frac{9+4\sqrt{5}}{10}\omega^t+\frac{1}{2}\frac{1}{2^t}-\frac{1}{10}\frac{1}{4^t}+k\Big(\boldsymbol{R}\frac{2+\sqrt{5}}{10}\omega^t-\frac{1}{2}\frac{1}{2^t}+\frac{3}{10}\frac{1}{4^t}\Big)\Big\},$$