135. *Probabilities on Inheritance in Consanguineous Families. IX*

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VIII. Combinations through the most extreme consanguineous marriages

1. Parents-descendants combinations

In the present and subsequence chapters, we shall supplement the results on combinations which have been postponed in VI, §1 as the extreme ones.

We first attempt to determine *the probability of parents-descendants combinations immediate after successive consanguineous marriages of the extreme mode, which will be designated by*

$$\mathfrak{e}_t(\alpha\beta, \gamma\delta; \xi_1\eta_1, \xi_2\eta_2) \equiv \varepsilon_{(11;0)_{t-1}|11}(\alpha\beta, \gamma\delta; \xi_1\eta_1, \xi_2\eta_2);$$

as to the notation cf. VI, §1.

It is readily seen that the quantity in consideration satisfies a recurrence equation

$$\mathfrak{e}_t(\alpha\beta, \gamma\delta; \xi_1\eta_1, \xi_2\eta_2) = \sum \varepsilon(\alpha\beta, \gamma\delta; ab, cd)\mathfrak{e}_{t-1}(ab, cd; \xi_1\eta_1, \xi_2\eta_2)$$

with

$$\mathfrak{e}_1(\alpha\beta, \gamma\delta; \xi_1\eta_1, \xi_2\eta_2) \equiv \varepsilon(\alpha\beta, \gamma\delta; \xi_1\eta_1, \xi_2\eta_2),$$

where the summation extends, as usual, over all the possible pairs of (ab, cd). Though the quantity $\mathfrak{e}_t$ has originally been defined for $t \geqq 1$, it is convenient to define $\mathfrak{e}_0$ as follows: When $(\alpha\beta, \gamma\delta)$ coincides with none of $(\xi_1\eta_1, \xi_2\eta_2)$ and $(\xi_2\eta_2, \xi_1\eta_1)$, then $\mathfrak{e}_0(\alpha\beta, \gamma\delta; \xi_1\eta_1, \xi_2\eta_2) = 0$; when $(\alpha\beta, \gamma\delta)$ coincides with $(\xi_1\eta_1, \xi_2\eta_2)$ or $(\xi_2\eta_2, \xi_1\eta_1)$, then $\mathfrak{e}_0 = 1$ or $\mathfrak{e}_0 = 1/2$ according to $A_{\alpha\beta} = A_{\gamma\delta}$ or $A_{\alpha\beta} \neq A_{\gamma\delta}$.

To determine the values of the $\mathfrak{e}_t$'s, we distinguish four systems according to the number of different genes contained in parents' types, based on a reason that possible types of descendants are restricted to those consisting of the genes involved in their parents' types. In fact, intermediate marriages under consideration are so extreme that there concern no individuals from other lineages.

In each system, the recurrence equation can be regarded, for a fixed pair of descendants' types as *a system of difference equations* of the first order in which the unknowns are the probabilities for possible pairs of parents' types. For the sake of brevity, we shall use for a while an abbreviation

$$[\alpha\beta, \gamma\delta]_t \equiv \mathfrak{e}_t(\alpha\beta, \gamma\delta; \xi_1\eta_1, \xi_2\eta_2),$$

for a fixed pair $(\xi_1\eta_1, \xi_2\eta_2)$, unless any confusion can arise.