119. On Newman Algebra. II

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Introduction

In our earlier paper "On Newman Algebra"[1] (we shall refer to the paper as I) we have considered the Newman algebra in a somewhat different sense from the usual one, and given a postulate-set for this algebraic system. Namely, in Newman algebra in the usual sense[2], the associative law for multiplication does not always hold, while this law holds in the system considered in I. We shall continue to use the term *Newman algebra* in the same sense as in I, and propose to give another postulate-set for the system. The postulate-set given in I will be denoted as Set I*, and the new set as Set II*. Set II* is obtained from Set I* by replacing the commutative law for addition

$$\text{I}^*.\ 3.\quad a+b=b+a$$

by a more restricted one

$$\text{II}^*.\ 8'.\quad a+b'b=b'b+a.$$

In §1 we shall give a complete list of postulates of Set II*, and prove the equivalence of Sets I* and II*. In I, we have obtained as a by-product the postulate-sets I and II characterizing respectively the Boolean lattice and the Boolean ring with unity. Correspondingly we shall obtain here new postulate-sets III and IV. In §2 we shall give a proof for the associative law for addition $(a+b)+c=a+(b+c)$, different from the one given in I, and in §3 independence proofs for our Set II* will be given. As the Sets I, II included the Set I* in I, so that the independence proofs for the Sets I, II implied that of Set I*, so the Sets III, IV include our Set II*, and the independence proofs for the Sets III, IV imply that of Set II*. Now the independence systems for the Sets I, II serve also as such for the Sets III, IV. But we shall show that we can construct the simpler independence systems for Postulate 5 of Sets III, IV than those given in I. These systems show also the independence of Postulate 5 of Sets I*, I, II.

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