116. *Note on Generalized Uniserial Algebras. II*

By Tensho YOSHII and Masatoshi IKEDA
Department of Mathematics, Osaka University, Osaka, Japan
(Comm. by K. SHODA, M.J.A., July 12, 1954)

Let A be an associative (and finite dimensional) algebra with a unit and K be a ground field. In the previous paper[1] one of the authors proved that the *absolutely generalized uniserial* algebras, i.e. the generalized uniserial algebras which remain so after any coefficient field extension, are the direct sum of two subalgebras, one of which is a generalized uniserial algebra with the separable residue class algebra over its radical and the other an absolutely uniserial algebra, and the converse is true.

In this note we shall prove the following two theorems:

Theorem 1. *If A_L is generalized uniserial for an extension field L of K, then A has the same property.*

Theorem 2. *If A_L has the radical expressible as a principal ideal, then A has the same property.*

1. **Proof of Theorem 1.** Let N be the radical of A. We may assume that $N^2=0$, since A is generalized uniserial if and only if A/N^2 is generalized uniserial.[2] Let $A=\sum_i Ae_i$ be a direct decomposition of A into directly indecomposable components, and $e_i=\sum_{j=1}^{r_i} f_j^{(i)}$ be the decomposition of the idempotent e_i into primitive orthogonal idempotents in A_L. Assuming

$$r_1=\cdots=r_{n_1}\lneqq r_{n_1+1}=\cdots=r_{n_2}\lneqq\cdots\lneqq r_{n_{\lambda-1}+1}=\cdots=r_n,$$

we can classify e_κ into

$$\mathfrak{E}_1=(e_1,\ldots, e_{n_1})$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\mathfrak{E}_\lambda=(e_{n_{\lambda-1}+1},\ldots, e_n).$$

Then $N_Le_i=\sum_{j=1}^{r_1} N_Lf_j^{(i)}$ $(e_i\in\mathfrak{E}_1)$, where $N_Lf_j^{(i)}$ is directly indecomposable by the assumption of A_L. On the other hand, if $Ne_i\simeq\sum_j\bar{A}\bar{e}_j$, where $\bar{A}\bar{e}_j=Ae_j/Ne_j$, then $N_Le_i\simeq\sum_j\bar{A}_L\bar{e}_j$ and the left hand side is decomposed into r_1 directly indecomposable components. Therefore by the Remak Schmidt Theorem, the right hand side has r_1 directly indecomposable components. But from the assumption of r_1 we have $N_Le_i\simeq\bar{A}_L\bar{e}_j$ and $Ne_i\simeq\bar{A}\bar{e}_j$ $(e_j\in\mathfrak{E}_1)$. Moreover $e_{j'}Ne_i=0$ for $e_{j'}\notin\mathfrak{E}_1$. Similarly $e_iN\simeq\bar{e}_k\bar{A}$ $(e_k\in\mathfrak{E}_1)$ for $e_i\in\mathfrak{E}_1$ and $e_iNe_{j'}=0$ for $e_{j'}\notin\mathfrak{E}_1$.

1) T. Yoshii [3].
2) T. Nakayama [2], Lemma 3.