114. On a Certain Type of Analytic Fiber Bundles

By Shigeo NAKANO

Mathematical Institute, Kyoto University

(Comm. by Z. SUETUNA, M.J.A., July 12, 1954)

In a lecture at University of Chicago (cf. [1]), A. Weil developed the theory of algebraic fiber varieties. Among others, he treated fiber varieties over a non-singular algebraic curve, which have the projective straight line as fibers and the group of affine transformations as the structure group. He classified these fiber varieties in a purely algebraic way (and in the case of a universal domain of any characteristic). In this note we shall show that his second invariant admits a simple and natural interpretation, as far as complex analytic fiber bundles are concerned.

1. Let V be a compact complex analytic manifold. A fiber bundle $\mathfrak{B}$ to be considered here is defined in terms of a finite open covering $\{U_j\}$ of V, and a system of holomorphic mappings s_{jk} from $U_j \frown U_k$ into G; the group of the affine transformations of a complex affine straight line $\boldsymbol{C}$. Here the mappings s_{jk} satisfy the relation

(1) $$s_{jk} \cdot s_{kl} = s_{jl} \qquad \text{in } U_j \frown U_k \frown U_l.$$

If we write

$$s_{jk} \cdot \zeta = a_{jk}\zeta + b_{jk} \qquad \text{for } \zeta \in \boldsymbol{C},$$

then a_{jk} and b_{jk} are holomorphic functions in $U_j \frown U_k$ and

(2) $$\begin{cases} a_{jk} \cdot a_{kl} = a_{jl} \\ a_{jk} \cdot b_{kl} + b_{jk} = b_{jl}, \end{cases}$$

while $\mathfrak{B}$ may be described in terms of "coordinates" (z, ζ_j) ($z \in U_j$ and $\zeta_j \in \boldsymbol{C}$), with the relation

(3) $$(z, \zeta_j) \sim (z', \zeta_k) \text{ if and only if } \begin{cases} z = z' \in U_j \frown U_k \\ \zeta_j = a_{jk}(z)\zeta_k + b_{jk}(z). \end{cases}$$

Two systems $s_{jk} = (a_{jk}, b_{jk})$ and $s'_{jk} = (a'_{jk}, b'_{jk})$ define the same bundle if and only if

$$s'_{jk} = t_j^{-1} s_{jk} t_k,$$

where each $t_j = (c_j, d_j)$ is a holomorphic mapping of U_j into G. In terms of a, b, c and d, this condition is expressed as

(4) $$\begin{cases} a'_{jk} = c_j^{-1} \cdot a_{jk} \cdot c_k \\ b'_{jk} = c_j^{-1}(a_{jk}d_k + b_{jk} - d_j). \end{cases}$$

If $\mathfrak{B}$ is defined by (a_{jk}, b_{jk}), then (2) shows that (a_{jk}) defines a complex line bundle $\mathfrak{A}$ (abbreviation: C.L.B.) in the sense of