157. *The Divergence of Interpolations.* I

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The convergence of interpolation polynomials to a given function in the points which satisfy a certain condition has been studied sufficiently by Walsh and others.

Let $f(z)$ be a function which is single valued and analytic throughout the interior of the circle $C_R: |z|=R>0$ and which has singularities on C_R. Let $W_n(z)$ be a sequence of polynomials of respective degrees n such that the sequence of $W_n(z)/z^n$ converges to $\lambda(z)$ analytic and non-vanishing exterior to a circle $C_{R'}: |z|=R'<R$ and uniformly on any closed limited point set exterior to $C_{R'}$. Then the sequence of polynomials $S_n(z;f)$ of respective degrees n found by interpolation to $f(z)$ in all the zeros of $W_{n+1}(z)$ converges to $f(z)$ uniformly on any closed set interior to C_R.

But the divergence of $S_n(z;f)$ at every point exterior to C_R is not yet established in general, as far as I know. If we choose a certain condition of $W_n(z)$ which is stronger than the condition above-mentioned, the divergence at every point exterior to C_R can be proved. (Cf. T. Kakehashi: On the convergence-region of interpolation polynomials, Journal of the Mathematical Society of Japan, **6**(1954).)

The purpose of this paper is to study the divergence of $S_n(z;f)$ which interpolate to $f(z)$ with singularities of a certain type on C_R, in the points which satisfy the condition mentioned formerly.

1. Let $\varphi(t)$ be the function single valued and analytic on the circle C_R: $|t|=R>0$, a be a point on C_R and m be a complex number. If the real part of m is positive, the integral

$$\int_{C_R} \varphi(t)\,(t-a)^{m-1}\,dt \quad ; \quad a=Re^{i\alpha}$$

exists. But if the real part of m is not positive, the above integral does not exist. For such cases, we define the finite part of the integral as follows:

$$\text{Pf.} \int_{C_R} \varphi(t)\,(t-a)^{m-1}dt=\int_{C_R} \psi(t)\,(t-a)^{m+p}\,dt, \tag{1}$$

where $\psi(t)$ is the function single valued and analytic defined by

$$\varphi(t)=\sum_{k=0}^{p} \frac{\varphi^{(k)}(a)}{k!}(t-a)^k+(t-a)^{p+1}\psi(t) \tag{2}$$