

179. A Note on the General Metrization Problem

By Kiyoshi ISÉKI

Kobe University

(Comm. by K. KUNUGI, M.J.A., Nov. 12, 1954)

A remarkable and important class of topological spaces was discovered by J. Dieudonné (3). He has called it paracompact space. An important result was obtained by A. H. Stone (9), who proved that every metric space is paracompact. The result of A. H. Stone led Yu. Smirnov (8) to find an interesting topological condition of the general metrization problem.¹⁾

Hereafter all spaces will be understood to be Hausdorff spaces. A family α of open sets is called σ -locally finite, if $\alpha = \bigcup_{i=1}^{\infty} \alpha_i$ such that each α_i is locally finite. Then the result obtained by Yu. Smirnov (8) is the following

Theorem. A necessary and sufficient condition that a regular topological space be metrizable is that it has an open σ -locally finite basis for the open sets.

In this Note, we shall give a simple proof of Theorem.

From the results of A. H. Stone, the necessity of Theorem is clear. We turn now to prove the sufficiency of the condition. We first summarize some results needed in the proof.

Let S be a regular topological space having a σ -locally finite basis for the open sets, then each subset of S is regular, and, since a theorem of E. A. Michael (7), S is paracompact and normal. Let G be an open set in S , and let x be a point of G , then there is an open set $O_n(x) \in \alpha_n$ for some n such that $x \in O_n(x) \subset \overline{O_n(x)} \subset G$. Let F_n be the closure of the union of O_n contained in α_n . Since the locally finiteness of α_n , F_n is the union of every closure of O_n . Therefore, we have $G = \bigcup_{n=1}^{\infty} F_n$. This shows that S is perfectly normal. (Hence, since theorems of E. A. Michael (7) and J. Dieudonné (3), every subset of S is paracompact and normal.)

The open covering of S consisting of α_n plus S has a locally finite refinement of open sets $\beta_n = \{U_{\alpha,n}\}$. Since S is perfectly normal, we can find the continuous decomposition $\{\varphi_{\alpha,n}(x)\}$ of unity satisfying the following conditions

$$(1) \quad \sum_{\alpha} \varphi_{\alpha,n}(x) = 1 \quad \text{and} \quad 0 \leq \varphi_{\alpha,n}(x) \leq 1,$$

1) The classical results and its proofs of the metrization problem was found in E. W. Chittenden (2), P. Lorenzen (6) and C. Kuratowski (5). On the other hand, R. H. Bing (1) has given a general condition by his new notions: If a regular topological space is perfectly screenable, then it is metrizable.