176. On Abhomotopy Group in Relative Case

By Yoshiro INOUE

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Introduction

The (n, r)-th abhomotopy group $\kappa_r^n(Y, y_0)$ of a space Y as base point $y_0 \in Y$ was defined by S. T. Hu as a generalization of Abe groups (M. Abe [1]). He showed that its algebraic structure is completely determined in terms of homotopy groups of Y, and that

$$(*) \qquad \kappa_r^n(Y, y_0) \approx \pi_{r+1}(Y^{S^{n-r-1}}, k_0) \quad r \geq 0,$$

where $Y^{S^{n-r-1}}$ is a mapping space consisting of all maps $f: S^{n-r-1} \to Y$ and topologized by compact open topology due to R. H. Fox (R. H. Fox [2]), and k_0 is a constant map: $k_0: S^{n-r-1} \to y_0$ (S. T. Hu[3]). In this paper, I shall show that the notion of abhomotopy group is relativized by using the same relation as $(*)$. In this paper, we always denote by Y a given topological space, by Y_0 a subspace of Y and y_0 a reference point of Y_0. Then the *(m, n)-th relative abhomotopy group* $\kappa_n^m(Y, Y_0, y_0)$ of (Y, Y_0, y_0) is defined by

$$(**) \qquad \kappa_n^m(Y, Y_0, y_0) = \pi_m(Y^{E^n}\{S^{n-1}, Y_0\}, k_0) \quad m, n \geq 1,$$

where $Y^{E^n}\{S^{n-1}, Y_0\}$ is a mapping space consisting of all maps f: $E^n, S^{n-1} \to Y, Y_0$ and topologized by compact open topology. I shall show that, in § 2, its algebraic structure is completely determined by $\pi_{m+n}(Y, Y_0, y_0)$ and $\pi_m(Y_0, y_0)$. In § 1, for a preliminary of § 2, I describe a definition of relative homotopy groups which is obtained by a slightly modification of that of absolute homotopy groups given in the book "S. T. Hu [4] § 21".

§ 1. **Preliminary.** 1.1. Let I^{n+1} be the $(n+1)$-cube, and $\dot{I}^{n+1}$ be the boundary of I^{n+1} as usual. We use the following notations:

$$I^n = \{x^{n+1} = (x_1, \ldots, x_{n+1}) \in I^{n+1} \mid x_{n+1} = 0\},$$
$$J^n = \dot{I}^{n+1} - I^n,$$
$$P_n^n = \{x^{n+1} = (x_1, \ldots, x_{n+1}) \in I^{n+1} \mid x_n = 0\},$$
$$x_0 = (0, \ldots, 0) \in \dot{I}^{n+1}.$$

Let $\mathfrak{F} = Y^{J^n}\{\dot{I}^n, Y_0; x_0, y_0\}$ be the totality of maps $f: J^n, \dot{I}^n, x_0 \to Y, Y_0, y_0$. The maps f of $\mathfrak{F}$ are divided into disjoint homotopy classes relative to $\{\dot{I}^n, Y_0; x_0, y_0\}$. Denote by Ω the totality of these classes and by $[f]$ the class containing $f \in \mathfrak{F}$. Let f be a representative of an arbitrary element α of $\pi_n(Y, Y_0, y_0)$. Define a map $\mu f: J^n \to Y$ by taking for each $x^{n+1} = (x_1, \ldots, x_{n+1}) \in J^n$