174. *Dirichlet Problem on Riemann Surfaces. III*

(*Types of Covering Surfaces*)

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Let $\underline{R}$ be a null-boundary Riemann surface and let R be a positive boundary Riemann surface given as a covering surface.

1) If $\mu(R, \mathfrak{A}(R, \underline{R}^*))=1$, we call R a covering surface of D-type over $\underline{R}$.

2) We map R^∞ onto the unit-circle $U_\xi: |\xi|<1$ conformally. If the composed function $\underline{z}=\underline{z}(\xi): U_\xi \to R \to \underline{R}^*$ has angular limits with respect to $\underline{R}$ almost everywhere on $|\xi|=1$. We call R a covering surface of F-type over $\underline{R}$.

3) Let $T(r)$ be the characteristic function of the mapping $R \to \underline{R}$. If $T(r)$ is bounded, we say, R is a covering surface of bounded type. By Theorem 1.1, it is easy to see that we have

Bounded type$\to$[1] F-type$\to D$-type, and that F-type implies $\mu(R^\infty, \mathfrak{A}(R^\infty, \underline{R}^*))=1$. If the universal covering surface of the projection of R is hyperbolic, $\mu(R^\infty, \mathfrak{A}(R^\infty, \underline{R}^*))=1$ implies that R is a covering surface of F-type, because $\mu(R^\infty, \mathfrak{A}(R^\infty, B))$[2] $=0$.

Let $\hat{R}$ be a covering surface over R. In the following, we investigate the relations between Riemann surface $\hat{R}$ and R. By Theorem 1.1 we have at once the following

Theorem 3.1. If R is a covering surface of bounded type, then $\hat{R}$ is also of bounded type relative to $\underline{R}$.

Theorem 3.2. Let R be a covering surface such that the universal covering surface of the projection $\underline{R}'^\infty$ of R is hyperbolic. We map $\underline{R}'^\infty$, R^∞ and $\hat{R}^\infty$ conformally onto the unit-circles $U_\xi: |\xi|<1$, $U_\eta: |\eta|<1$ and $U_\zeta: |\zeta|<1$ respectively. Let $\eta=\eta(\zeta)$, $\xi=\xi(\zeta)$ and $\xi=\xi(\eta)$ be mappings $U_\zeta \to U_\eta$, $U_\zeta \to U_\xi$ and $U_\zeta \to U_\xi$ respectively. Then we have

$$\mu(\hat{R}^\infty, \mathfrak{A}(\hat{R}^\infty, \underline{R}^*)) \geqq \mu(R^\infty, \mathfrak{A}(R^\infty, \underline{R}^*)).$$

Proof. Since $\mu(\underline{R}'^\infty, \mathfrak{A}(\underline{R}'^\infty, B))=\mu(R^\infty, \mathfrak{A}(R^\infty, B))=\mu(\hat{R}^\infty, \mathfrak{A}(\hat{R}^\infty, B))=0$ without loss of generality, we can suppose that every A.B.P. lies on $\underline{R}$. Let A_η and A_ζ be images of $\mathfrak{A}(R^\infty, \underline{R})$ and $\mathfrak{A}(\hat{R}^\infty, \underline{R})$ respectively, and let ${}_\eta S_\zeta$, ${}_\xi S_\zeta$ and ${}_\xi S_\eta$ be the sets where the corresponding functions

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1) $\to$ means implication.

2) Measure of a set of A.B.P.'s of R^∞ with projections on the ideal boundary B of $\underline{R}$.