# 173. *Dirichlet Problem on Riemann Surfaces.* II

## (*Harmonic Measures of the Set of Accessible Boundary Points*)

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Let $\underline{R}$ be a null-boundary Riemann surface with $A$-topology[1) and let $R$ be a positive boundary Riemann surface given as a covering surface over $\underline{R}$. When a curve $L$ on $R$ converges to the boundary of $R$ and its projection $\underline{L}$ on $\underline{R}$ tends to a point of $\underline{R}^*$, we say that $L$ determines an accessible boundary point (A.B.P.) relative to $\underline{R}^*$. In the following we denote the set of all A.B.P.'s by $\mathfrak{A}(R, \underline{R}^*)$. We consider continuous super-harmonic function $v(z)$ in $R$ such that $0 \leqq v(z) \leqq 1$ and $\lim v(z)=1$ when $z$ tends to the boundary along every curve determining an A.B.P. and we denote by $\mu(R, \mathfrak{A}(R, \underline{R}^*))$ the lower envelope of above functions which is harmonic in $R$ on account of Perron-Brelot's theorem. We also consider $\mathfrak{A}(R^\infty, \underline{R}^*)$ and $\mu(R^\infty, \mathfrak{A}(R^\infty, \underline{R}^*))$ defined similarly on $R^\infty$. In the following we assume that the universal covering surface of the projection of $R$ on $\underline{R}$ is hyperbolic. Then there exists a null-boundary Riemann surface $\underline{R}'$ such that the projection of $R \subset \underline{R}'$, $\underline{R}' \subset \underline{R}$ and that $\underline{R}'^\infty$ is hyperbolic. We map $\underline{R}'^\infty$ and $R^\infty$ conformally onto $U_\eta : |\eta|<1$ and $U_\xi : |\xi|<1$ respectively. Let $l_\xi$ be a curve in $U_\xi$ determining an A.B.P. of $R^\infty$, whose projection on $\underline{R}'$. Then we see that $l_\xi$ converges to a point $\xi_0 : |\xi_0|=1$ and $\underline{z}=\underline{z}(\xi) : U_\xi \to R \to \underline{R}'$ has an angular limit at $\xi_0$. It follows that $\underline{z}=\underline{z}(\xi)$ has angular limits at every point of $A'_\xi$ with respect to $\underline{R}'$, where $A'_\xi$ is the set of points $\xi'$ on $|\xi|=1$ such that at least one curve determining A.B.P. with projection in $\underline{R}$ terminates at $\xi'$.

Let $\{R'_\lambda\}$ be an exhaustion of $\underline{R}'$ and $\Delta_{l,m,n}(\theta)$ be the set such that $\frac{1}{n} \leqq |\xi - e^{i\theta}| < \frac{1}{m}$ and $|\arg(1-e^{-i\theta}\xi)| < \frac{\pi}{2} - \frac{1}{l}$ and let $\delta(f(\xi))$ be the diameter of the set $f(\xi) : \xi \in \Delta_{l,m,n}(\theta)$ with respect to the $A$-topology. Then we have

$$A'_\xi = \mathcal{E}_{\theta}\Big[\sum_{\lambda} \prod_{l} \prod_{k} \sum_{m} \prod_{n} \delta(f(\xi)) \leqq \frac{1}{k} \leftarrow \xi \in \Delta_{l,m,m+n}(\theta)\Big].$$

Since $\delta(f(\xi))$ is continuous with respect to $\theta$ for fixed $l$, $m$ and $n$, this shows that $A'_\xi$ is a Borel set.

M. Ohtsuka has proved the next

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1) See, Dirichlet problem. I.