170. *Uniform Convergence of Fourier Series. III*

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1. Introduction. S. Izumi and G. Sunouchi[1] proved the following theorems concerning uniform convergence of Fourier series:

Theorem I. *If*

$$f(t)-f(t')=o\left(1\Big/\log\frac{1}{|t-t'|}\right) \text{ as } t, t'\to x$$

then the Fourier series of $f(t)$ converges uniformly at $t=x$.

Theorem II. *If*

$$f(t)-f(t')=o\left(1\Big/\log\log\frac{1}{|t-t'|}\right) \text{ as } t, t'\to x$$

and the nth Fourier coefficients are $O((\log n)^{\alpha}/n)$ for $\alpha>0$, then the Fourier series of $f(t)$ converges uniformly at $t=x$.

In this paper, we treat the case that the order of $f(t)-f(t')$ is $o\left(1\Big/\left(\log\frac{1}{|t-t'|}\right)^{\alpha}\right)$ $(1>\alpha>0)$, $o\left(1\Big/\left(\log\log\frac{1}{|t-t'|}\right)^{\alpha}\right)$ $(\alpha>0)$ and more generally $o\left(1\Big/\left(\log_k\frac{1}{|t-t'|}\right)^{\alpha}\right)$.

2. Theorem 1. *Let $0<\alpha<1$. If*

$$f(t)-f(t')=o\left(1\Big/\left(\log\frac{1}{|t-t'|}\right)^{\alpha}\right) \quad (t, t'\to 0)$$

and the nth Fourier coefficients of $f(t)$ is of order $O(e^{(\log n)^{\alpha}}/n)$, then the Fourier series of $f(t)$ converges uniformly at $t=0$.

Proof. We assume that $x_n\to 0$ and $f(0)=0$.

$$\begin{aligned} S_n(x_n)&=\frac{1}{\pi}\int_0^{\pi}[f(x_n+t)+f(x_n-t)]\frac{\sin nt}{t}dt+o(1)\\ &=\frac{1}{\pi}\left[\int_0^{\pi/n}+\int_{\pi/n}^{\pi e^{\beta(\log n)^{\alpha}}/n}+\int_{\pi e^{\beta(\log n)^{\alpha}}/n}^{\pi}\right]+o(1)\\ &=\frac{1}{\pi}[I+J+K]+o(1), \end{aligned}$$

say, where β is the least number >1 such that $2n\,|\,e^{\beta(\log n)^{\alpha}}$, then it is sufficient to prove that $s_n(x_n)=o(1)$ as $n\to\infty$.

Since $f(x)$ is continuous, we have $I=o(1)$.

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1) S. Izumi and G. Sunouchi: Notes on Fourier analysis (XLVIII): Uniform convergence of Fourier series, Tôhoku Mathematical Journal, **3** (1951).