168. *On the Strong Summability of the Derived Fourier Series*

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1. Let $f(t)$ be a periodic function of bounded variation with period 2π, and its Fourier series be

$$a_0/2+\sum_{n=1}^{\infty}(a_n\cos nt+b_n\sin nt)=\sum_{n=0}^{\infty}A_n(t).$$

We shall consider the derived Fourier series

$$\sum_{n=1}^{\infty}n(b_n\cos nt-a_n\sin nt)=\sum_{n=1}^{\infty}A_n'(t)$$

and its conjugate series

$$\sum_{n=1}^{\infty}n(a_n\cos nt+b_n\sin nt)=\sum_{n=1}^{\infty}B_n'(t).$$

We denote by $\tau_n(t)$ and $\bar{\tau}_n(t)$ the nth partial sums of them, i.e.

$$\tau_n(t)=\sum_{m=1}^{n}m(b_m\cos mt-a_m\sin mt)=\sum_{m=1}^{n}A_m'(t),$$

$$\bar{\tau}_n(t)=\sum_{m=1}^{n}m(a_m\cos mt+b_m\sin mt)=\sum_{m=1}^{n}B_m'(t).$$

As in the case of Fourier series, we use the modified partial sums of them;

$$\tau_n^*(t)=\tau_n(t)-A_n'(t)/2,\quad \bar{\tau}_n^*(t)={}_n\bar{\tau}(t)-B_n'(t)/2.$$

Recently B. N. Prasad and U. N. Singh[1] proved the following theorems:

Theorem A. *If $f(t)$ is a continuous function of bounded variation which is differentiable at $t=x$ and if for some $\varepsilon>0$*

$$G(t)=\int_0^t|dg(u)|=o\left\{t\left(\log\frac{1}{t}\right)^{-1-\varepsilon}\right\},\ \text{as } t\to 0,$$

where $g(u)=g_x(u)=f(x+u)-f(x-u)-2uf'(x)$, then

$$\sum_{m=1}^{n}|\tau_m(x)-f'(x)|=o(n).$$

That is, the derived Fourier series of $f(t)$ is $(H,1)$ summable to the sum $f'(x)$ at $t=x$.

Theorem B. *If $f(t)$ is a continuous function of bounded variation which is differentiable at $t=x$ and if for some $\varepsilon>0$*

1) B. N. Prasad and U. N. Singh: Math. Zeits., **56**, 280-288 (1952).