166. *The Minimum Area of Convex Curves for Given Diameter and Perimeter*

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§1. Among the quantities concerning ovals the following are of fundamental importance: the area F, the perimeter L, the diameter D and the thickness Δ. Various relations between these quantities have been investigated by Kubota and others. However some of them, so-called minimum problems of a certain kind, remain unsolved. Already we, Kubota and I, solved two problems[1] of them as follows:

$$F \geqq 3\Delta\left\{\sqrt{D^2-\Delta^2}+\Delta\left(\sin^{-1}\frac{\Delta}{D}-\frac{\pi}{3}\right)\right\}-\frac{\sqrt{3}}{2}D^2 \quad \text{for } D \geqq \Delta \geqq \sqrt{3}D/2,$$

$$2F \geqq \Delta L-\sqrt{3}\,\Delta^2 \sec^2\theta \qquad \text{for } \pi\Delta \leqq L \leqq 2\sqrt{3}\,\Delta,$$

where θ is the root of $\tan\theta-\theta=(L-\pi\Delta)/(6\Delta)$ in the interval $0 \leqq \theta \leqq \pi/6$.

Before printing our results similar studies[2][3] were published in Germany and U.S.A. About that time Prof. T. Kubota died who often gave me kind advices and was my joint worker. The publication of our paper was delayed as we could not solve the (L, D) problem: the problem of the minimum figures for F when D and L are so given that $3D < L < \pi D$. In the two papers above-mentioned, the former did not refer to the (L, D) problem and the latter, M. Sholander's paper, gave the partial results for this problem and concluded as follows: "It is now natural to conjecture that the minimum figure is a triarc RST in the form of polygon inscribed in the Reuleaux triangle RST. Assuming the truth of conjecture, a much more accurate description of the figure can be given. It remains doubtful, however, whether for $3D < L < \pi D$ a simple inequality giving lower bounds for F in terms of L and D exists which is better than Kubota's inequality". On the other hand I had

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1) T. Kubota and D. Hemmi: Some problems of minima concerning ovals, J. Math. Soc. Japan, which was read at the annual meeting of the Math. Soc. of Japan held in June 2, 1951.

2) D. Ohmann: Extremalprobleme für konvexe Bereiche der euklidischen Ebene, Math. Z., **55**, 347-352 (1952).

3) M. Sholander: On certain minimum problems in the theory of convex curves, Trans. Amer. Math. Soc., **73**, 139-173 (1952).