203. The Divergence of Interpolations. III

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4. Next we consider a generalization of Theorem 4. Let D be a closed limited points set whose complement K with respect to the extended plane is connected and regular in the sense that K possesses a Green's function with pole at infinity. Let $w=\phi(z)$ map K onto the region $|w|>1$ so that the points at infinity correspond to each other. Let $\Gamma_R(R>1)$ be the level curve determined by $|w|=R>1$.

In this case, we can also define the operation Y_m to $\varphi(z)$ single valued and analytic on Γ_R by

$$(22)\quad \begin{cases} Y_m(\varphi\,;\,a)=\dfrac{\Gamma(1-m)}{2\pi i}\,\text{pf.}\displaystyle\int_{\Gamma_R}\varphi(t)[\phi(t)-\phi(a)]^{m-1}dt; \\ \qquad\qquad\qquad\qquad a \text{ on } \Gamma_R;\ m\neq 1,\ 2,\ldots, \\ Y_m(\varphi\,;\,a)=\dfrac{(-1)^m}{2\pi i}\displaystyle\int_{\Gamma_R}\varphi(t)L_m[\phi(t)-\phi(a)]dt; \\ \qquad\qquad\qquad\qquad a \text{ on } \Gamma_R;\ m=1,\ 2,\ldots, \end{cases}$$

where $(w-\phi(a))^{m-1}$ and $L_m(w-\phi(a))$ are functions single valued and analytic interior to Γ_R which are defined in paragraph 1.

Given a function $f(z)$ which is single valued and analytic throughout the interior of the level curve Γ_R and which has singularities of Y_m type on Γ_R, that is

$$(23)\qquad f(z)=g(z)+\textstyle\sum_{k=1}^{N}g_k(z)y_{m_k}(\phi(z)\,;\,a_k);\quad a_k \text{ on } \Gamma_R,$$

where $g(z)$ and $g_k(z)$ are functions defined by (8) which are single valued and analytic on and within the level curve Γ_R, and $y_{m_k}(w\,;\,a_k)$ are functions defined by (8) which are single valued and analytic interior to Γ_R but have respectively a singularity of Y_m type at $z=a_k$.

Let a set of points (17) lie on D and satisfy the condition that the sequence $W_n(z)/\Delta^n w^n$ converges to an analytic function $\lambda(w)=\lambda(\phi(z))$ non-vanishing for z exterior to D, and uniformly on any closed limited points set exterior to D, and uniformly on any closed limited points set exterior to D, where Δ is *capacity* of D. That is, for any positive number greater than unity,

$$(24)\qquad \lim_{n\to\infty} W_n(z)/\Delta^n w^n=\lambda(w)\neq 0 \text{ uniformly for } |w|\geqq r>1.$$

The sequence of polynomials $S_n(z\,;f)$ of respective degrees n found by interpolation to $f(z)$ in all the zeros of $W_{n+1}(z)$ is defined by