200. *Dirichlet Problem on Riemann Surfaces. IV*
(*Covering Surfaces of Finite Number of Sheets*)

By Zenjiro KURAMOCHI
Mathematical Institute, Osaka University
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Let $\underline{R}$ be a null-boundary Riemann surface with A-topology and let R be a covering surface over $\underline{R}$ and let L be a curve in R determining an accessible boundary point (A.B.P.) $\mathfrak{p}$ with projection p. Denote by $V_n(p)$ the neighbourhood of p with diameter $\frac{1}{n}$ and denote by $\mathfrak{B}_n$ the set of R lying over $V_n(p)$, which is composed of at most enumerably infinite number of domains $D^i_n(p)$ $(i=1, 2, \ldots)$.

Associated domain. Let $D^i_n(\mathfrak{p})$ be a domain, over $V_n(p)$, containing an endpart of L. Two arcs L_1 and L_2 determine the same A.B.P., if and only if, for any number n, two associated domains of L_1 and L_2 are the same. This definition of A.B.P. is clearly equivalent to that of O. Teichmüller. Denote by $n(\underline{z}) : \underline{z} \in \underline{R}$ the number of times when $\underline{z}$ is coverd by R. Then it is clear that $n(\underline{z})$ is lower semi-continuous. When $\overline{\lim\limits_{z \in \underline{R}}}\, n(\underline{z}) > 1$, non accessible boundary points are complicated and in our case, it is sufficient to consider only $\mathfrak{A}(R, \underline{R})$, where $\mathfrak{A}(R, \underline{R})$ is the set of all A.B.P.'s.

Barrier. Let $B(z) : z \in R$ be a function such that $B(z)$ is non negative continuous super-harmonic function and that $\lim\limits_{z \to \mathfrak{p}} B(z) = 0$ and moreover for every associated domain $D_m(\mathfrak{p})$, there exists a number δ_m with the property that $\inf\limits_{z \notin D_m(\mathfrak{p})} B(z) > \delta_m (\delta_m > 0)$. We call $B(z)$ a barrier at $\mathfrak{p}$. It is well known that $\mathfrak{p}$ is regular for Dirichlet problem of R, if and only if, a barrier exists at $\mathfrak{p}$, under the condition that R is a covering surface of D-type over $\underline{R}$.

Lemma. Let R be a covering surface of D-type over $\underline{R}$ and let $\mathfrak{p}$ be an A.B.P. and let $D_n(\mathfrak{p})$ be an associated domain of $\mathfrak{p}$. We denote by proj $D_n(\mathfrak{p})$ the projection of $D_n(\mathfrak{p})$. If proj $\mathfrak{p}$ is regular for proj $D_n(\mathfrak{p})$, then $\mathfrak{p}$ is regular with respect to R.

In fact, let $B(\text{proj } \mathfrak{p})$ be a barrier of proj $\mathfrak{p}$ with respect to proj $D_n(\mathfrak{p})$. Then there exists a number δ such that $B(\text{proj } z) > \delta$, when $\underline{z} \notin$ proj $D_m(\mathfrak{p})$, where $m > n$, for given $D_m(\mathfrak{p})$. Put $B(z) = \text{Min}(\delta, B(\text{proj } z))$ in $D_m(\mathfrak{p})$ and $B(z) = \delta$ in $R - D_m(\mathfrak{p})$. Then $B(z)$ is clearly a barrier of $\mathfrak{p}$ with respect to R. Thus we have at once the following.

1) See, "Dirichlet problem. III".