194. Gentzen's Theorem on an Extended Predicate Calculus[1]

By Shôji MAEHARA
Mathematical Institute, University of Tokyo
(Comm. by Z. SUETUNA, M.J.A., Dec. 13, 1954)

In this paper, I shall show that the corresponding theorem (cf. 2) to Gentzen's 'Hauptsatz'[2] on his 'Kalkül LK' is proved in case of the logical system — I shall call it L_0K — obtained from LK by additional admitting quantifiers $V^0\varphi$ (for all φ) and $\exists^0\varphi$ (there exists φ) where φ is a propositional variable.

1. THE LOGICAL SYSTEM L_0K

1.1. 'Formula'

As the *definition* of formula, we use the *formation rule of formula* obtained from that of the *restricted predicate calculus* — for example, Kalkül LK — by adding to the latter the following item: If $\mathfrak{F}(\alpha)$ is a *formula*, α is a *free propositional variable without argument*, φ is a *bound propositional variable* not contained in $\mathfrak{F}(\alpha)$, and $\mathfrak{F}(\varphi)$ is the result of substituting φ for α throughout $\mathfrak{F}(\alpha)$, then $V^0\varphi\mathfrak{F}(\varphi)$ and $\exists^0\varphi\mathfrak{F}(\varphi)$ are *formulae*.

The *grade* of a formula is the number ($\geqq 0$) of occurrences of logical symbols (&, $\vee$, $\supset$, $\neg$, V, $\exists$, V^0, $\exists^0$) in the formula, and the *degree* the number of occurrences of V^0 and $\exists^0$. For example, a formula of the restricted predicate calculus has the degree 0.

1.2. 'Sequent'

A *sequent* is a formal expression of the form

$$\mathfrak{A}_1, \ldots, \mathfrak{A}_\mu \to \mathfrak{B}_1, \ldots, \mathfrak{B}_\nu$$

where $\mu, \nu \geqq 0$ and $\mathfrak{A}_1, \ldots, \mathfrak{A}_\mu, \mathfrak{B}_1, \ldots, \mathfrak{B}_\nu$ are arbitrary formulae. The part $\mathfrak{A}_1, \ldots, \mathfrak{A}_\mu$ is called the *antecedent*, and $\mathfrak{B}_1, \ldots, \mathfrak{B}_\nu$ the *succedent* of the sequent.

1.3. 'Rules of inference'

As *rules of inference* we use ones obtained from those for Gentzen's LK, which are represented as the 'Schlussfiguren-Schemata', by adding the following

Additional rules of inference for L_0K

1) Mr. G. Takeuti had proved otherwise the same result, and afterwards the present proof was obtained.

2) G. Gentzen: Untersuchungen über das logische Schliessen, Math. Zeitschr., **39**, 176–210, 405–431 (1935).