7. Harmonic Measures and Capacity of Sets of the Ideal Boundary. II

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Let R be a positive boundary Riemann surface and let D[1] be a non compact domain determining a subset B_D of the ideal boundary. Put $D_n=(R-R_n)\cap D$. Let $U_{n,n+i}(z)$ be a harmonic function in $R_{n+i}-R_0-D_n$ such that $U_{n,n+i}(z)=0$, on ∂R_0, $U_{n,n+i}(z)=1$ on ∂D_n and $\frac{\partial U_{n,n+i}}{\partial n}=0$ on $\partial R_{n+i}-D_n$. Then $\lim_{n=\infty}\lim_{i=\infty} U_{n,n+i}(z)=\lim_{n=\infty} U_n(z)=U(z)$, where $U(z)$ is the equilibrium potential of B_D. We have proved that

$$\int_{\partial R_0}\frac{\partial U_n}{\partial n}ds=\int_{\partial G_\varepsilon}\frac{\partial U_n}{\partial n}ds \tag{1}$$

for every G_ε except for at most one ε, where G_ε is the domain in which $U_n(z)>1-\varepsilon$. Let $U'_{n,n+i}(z)$ be a harmonic function in $R_{n+i}-G_\varepsilon-R_0$ such that $U'_{n,n+i}(z)=0$ on ∂R_0, $U'_{n,n+i}(z)=1-\varepsilon$ on $\partial G_\varepsilon\cap R_{n+i}$ and $\frac{\partial U'_{n,n+i}}{\partial n}=0$ on $\partial R_{n+i}-G_\varepsilon$. Then $\lim_{i=\infty} U'_{n,n+i}(z)=U_n(z)$. Since every $U'_{n,n+i}(z)=1-\varepsilon$ on ∂G_ε, $\frac{\partial U'_{n,n+i}}{\partial n}\to\frac{\partial U_n}{\partial n}$: $\frac{\partial U'_{n,n+i}}{\partial n}\leqq 0$ on every point of $\partial G_\varepsilon\cap R_{n+i}$. Hence by (1) and $\lim_{i=\infty}\int_{\partial R_0}\frac{\partial U_{n,n+i}}{\partial n}ds=\int_{\partial R_0}\frac{\partial U_n}{\partial n}ds$, we easily that

$$\lim_{i=\infty}\int_{\partial G_\varepsilon}\varphi_i\frac{\partial U'_{n,n+i}}{\partial n}ds=\int_{\partial G_\varepsilon}\varphi\frac{\partial U_n}{\partial n}ds \tag{2}$$

on ∂G_ε for every bounded sequence of continuous functions $\varphi_i\to\varphi$: $|\varphi_i|\leqq M<\infty$.

We denote by G_n the domain in which $U_n(z)>1-\varepsilon_n$, where $\varepsilon_1>\varepsilon_2>\cdots$; $\lim_n \varepsilon_n=0$ and every ε_n satisfies the condition (1).

Let $U''_{n,n+i}(z)$ be a harmonic function in $R_{n+i}-R_0-G_n$ such that $U''_{n,n+i}(z)=U(z)$ on $\partial G_\varepsilon+\partial R_0$ and $\frac{\partial U''_{n,n+i}}{\partial n}=0$ on $\partial R_{n+i}-G_n$. Since $U_n(z)$ is the function such that $U_n(z)=1-\varepsilon_n$ and $U_n(z)$ has the minimum Dirichlet integral over $R-R_0-G_n$, and since $\lim_{n=\infty} U_n(z)=U(z)$ on ∂G_n, then by (2) we can prove as in the previous paper[2]

$$\lim_{n=\infty}\lim_{i=\infty} U''_{n,n+i}(z)=U(z).$$

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1) See, the definition of non compact domain. "Harmonic measures and capacity. I".

2) See (1).