6. *Dirichlet Problem on Riemann Surfaces.* V
(*On Covering Surfaces*)

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(Comm. by K. KUNUGI, M.J.A., Jan. 12, 1955)

1) *Covering Surfaces over a Null-Boundary Riemann Surface*

Let $\underline{R}$ be a null-boundary Riemann surface and let R be a covering surface of F-type over $\underline{R}$. We denote by $\mathfrak{A}(R, \underline{R}^*)$ the set of all A.B.P.'s of R. Let $\mathfrak{F}$ be a closed set of $\mathfrak{A}(R, \underline{R}^*)$. The upper class $U^R_{\mathfrak{F}}$ is the set of all non negative continuous super-harmonic functions $U(z)$ such that $\lim U(z) \geqq 1$ along every curve tending to $\mathfrak{F}$. We denote by $\overline{H}^R_{\mathfrak{F}}(z)$ the lower envelope of $U^R_{\mathfrak{F}}$. Similarly the lower class $B^R_{\mathfrak{F}}$ is the class of all bounded continuous sub-harmonic functions $V(z)$ such that $\lim V(z) \leqq 0$ along every curve tending to the boundary except $\mathfrak{F}$. Further it is clear that $\overline{H}^R_{\mathfrak{F}}(z) \geqq \underline{H}^R_{\mathfrak{F}}(z)$ on a covering surface of D-type. If they coincide at one point of R. Then they are identical.

Lemma. Let $\mathfrak{F}$ be a closed set of $\mathfrak{A}(R, \underline{R}^*)$ of a covering surface of F-type. Then

$$\overline{H}^R_{\mathfrak{F}}(z) = \underline{H}^R_{\mathfrak{F}}(z).$$

Proof. We map the universal covering surface R^∞ of R onto the unit circle $U_\xi : |\xi| < 1$. Since by assumption, the mapping function $f(\xi): R^\infty \to R + \mathfrak{A}(R, \underline{R}^*)$ has angular limits almost everywhere on $|\xi| = 1$ and since $\mu(R, B) = 0$, where $\mu(R, B)$ is the outer harmonic measure of the boundary of R lying on the boundary of $\underline{R}$. We can suppose that $f(\xi)$ has angular limits lying in $\underline{R}$. Let $\mathfrak{F}'_n$ be the set of points of $R + \mathfrak{A}(R, \underline{R}^*)$ which have distance $\leqq \frac{1}{n}$ from $\mathfrak{F}$. Put $\mathfrak{F}_n = \mathfrak{F}'_n \cap \mathfrak{A}(R, \underline{R}^*)$ and let F be the regular image[1] of $\mathfrak{F}$. Then, since R is a covering surface of F-type, F_n is measurable and $f(\xi)$ has angular limits at F'_n, where $\text{mes}\,|F_n - F'_n| = 0$. Thus we can suppose $f(\xi)$ has angular limits at F_n. Let $\{R_m\}$ be an exhaustion of R with compact relative boundaries $\{\partial R_m\}$ and let $\partial \mathfrak{F}'_n$ be the relative boundary of $\mathfrak{F}'_n$. Let $\omega^n_{m,m+i}(z)$ be a harmonic function in $R_{m+i} - (\mathfrak{F}'_n \cap (R_{m+i} - R_m))$ such that $\omega^n_{m,m+i}(z) = 1$ on $\partial(\mathfrak{F}'_n \cap (R_{m+i} - R_m))$ and $\omega^n_{m,m+i}(z) = 0$ on $\partial R_{m+i} - \mathfrak{F}'_n$. Then we see that $\omega^n_{m,m+i}(z) \uparrow \omega^n_m(z)$ and $\omega^n_m(z) \downarrow \omega^n(z)$. Thus $\omega^n(z)$ is contained in the class $U^R_{\mathfrak{F}}$ for

1) See, "Dirichlet problem. III".