15. Some Remarks on Abhomotopy Groups

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1. Introduction. Abhomotopy groups has been introduced by S. T. Hu as a generalization of Abe groups (S. T. Hu [5]). Our purpose of the present paper is to show that abhomotopy groups can be treated as a special case of homotopy groups of pseudo fibre spaces. In the preceding paper [7], I defined abhomotopy groups of relative case. In latter part of this paper, I shall show that this groups is treated by the same method as above.

2. Pseudo Fibre Spaces. By a pseudo fibre space (X, p, B), we understand a collection of two spaces X, B and a continuous mapping $p: X\rightarrow B$ of X onto B which satisfy the "Lifting homotopy theorem" (p. 63, P. J. Hilton [3]; p. 443, J. P. Serre [8]). In this paper, we shall use the "Proposition 1" in p. 443 of J. P. Serre [8], which is equivalent to the "Lifting homotopy theorem". We recall that the homotopy sequence of a pseudo fibre space (X, p, B):

$$(1)\quad \longrightarrow \pi_{n+1}(B, b_0)\xrightarrow{d_{n+1}}\pi_n(X_0, x_0)\xrightarrow{i_n}\pi_n(X, x_0)\xrightarrow{p_n}\pi_n(B, b_0)\longrightarrow\cdots,$$

$n\geq 1$, is exact, where b_0 is a point of B, and x_0 is a point of the fibre $X_0=p^{-1}(b_0)$ over b_0. In the sequel, we shall use these notations in these senses.

3. T-Operators. In the remainder of this paper, we assume that the total space X of a pseudo fibre space (X, p, B) is arcwise connected. J. P. Serre has proved in his paper [8] that $\pi_1(B)$ operate on the homology groups of the fibre X_0. By the same method, $\pi_1(X)$ operate on the homotopy groups of X_0. First, we prove the following theorem.

Theorem 1. Let (X, p, B) be a pseudo fibre space, x be a point of X and X_x be the fibre over $p(x)\in B$. Then, the collection of the n-th homotopy groups $\{\pi_n(X_x, x)\mid x\in X\}$ form a local system of groups in the space X. (For the definition of a local system of groups, refer to §13; S. T. Hu [6].)

(Proof) Let $\sigma: I\rightarrow X$ be a path joining two points x_0 and x_1. Let $f: I^n\rightarrow X$ be a map of an element α of $\pi_n(X_{x_1}, x_1)$. Define a map $F: I^n\times 0\smile \dot{I}^n\times I\rightarrow X$ by taking for each $x^n\in I^n$, $t\in I$

$$F(x^n, t)=\begin{cases} f(x^n) & \text{on} \quad I^n\times 0 \\ w(1-t) & \text{on} \quad \dot{I}^n\times I. \end{cases}$$

Then the map $G=pF: I^n\times 0\smile \dot{I}^n\times I\rightarrow B$ has the extention $G': I^n\times I\rightarrow B$ defined by $G'(x^n, t)=p\omega(1-t)$. By the "Proposition 1" in p. 443